O. NEUGEBAUER AND D. PINGREE

THE PAÑCASIDDHĀNTIKĀ OF VARĀHAMIHIRA

PART II

Det Kongelige Danske Videnskabernes Selskab
Historisk-Filosofiske Skrifter 6, 1, Part II

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PREFACE

In our commentary to the first sixteen chapters we follow the order of the verses of the text, though often combined to form larger groups according to contents. In Chapter XVII, however, which is concerned with planetary theory based on Babylonian methods, we found it necessary to rearrange the material. The following Table of Contents will show the details.

For a number of technical terms for which no short equivalent exists in western astronomy (e.g., ahargaṇa or tithi) we give an alphabetical index on p. 129. An extensive index verborum will be found in Part I, p. 188 ff. The notation which we generally adopted in formulae is shown in the list on p. 130, arranged according to topics. Since numerical parameters provide one of the most powerful tools in the investigation of the interconnection between different sources, we have compiled a detailed index of parameters (p. 131 ff.).

A discussion of the general historical position of the Pañcasiddhāntikā in Indian astronomy is given in Part I.

O.N., D.P.

PART II

COMMENTARY

BY

O. NEUGEBAUER AND D. PINGREE

TABLE OF CONTENTS TO PART II

Chapter I

I,1. Varāhamihira's invocatory verses normally mention the Sun; cf. e.g. Bṛhatsaṃhitā I,1. This is appropriate because of his position as a Maga Brāhmaṇa. But here the Sun is also representative of the divine authors of siddhāntas (joined in the Pañcasiddhāntikā by Pitāmaha = Brahma), while Vasiṣṭha, who is one of the seven Vedic ṛṣis who form the constellation Saptarṣi (Ursa maior), represents the human authors (joined here by Pauliśa and Romaka). Varāhamihira's father, as stated Part I p. 7, was named Ādityadāsa ("Slave of the Sun").

I,2. The term bīja normally is applied only to a correction to a planet's mean longitude. It is not clear what in this verse is ascribed to "former teachers," or what is regarded as secret.

I,3. For the five siddhāntas and Lāṭadeva see Part I pp. 9–15.

I,4. The ranking in accuracy of the five siddhāntas given in this verse is appropriate: Sūrya, Pauliśa and Romaka, Vāsiṣṭha and Paitāmaha.

In the late seventh or during the eighth century (between 628, the date of the Brāhmasphuṭasiddhānta, and ca. 850, the date of the Prakaṭārthadīpikā), another verse based on this one became popular. It is quoted by Govindasvāmin in his Prakaṭārthadīpikā on II,8 of the Uttarabhāga of the Bṛhatpārāśarahorāśāstra:

> spaṣṭo brāhmas tu siddhāntas tasyāsannas tu romakaḥ/
> sauraḥ spaṣṭataro spaṣṭau vāsiṣṭhaḥ pauliśaś ca tau//

"The Brāhma (sphuṭasiddhānta) is accurate; the Romaka is close to it; the Sūrya (saura) is more accurate; and the Vāsiṣṭha and Pauliśa are inaccurate."

I,5–7. Varāhamihira here lists the contents of the Pañcasiddhāntikā, though in a fashion that is uncharacteristically unsystematic.

a) solar eclipses: VII; VIII; IX; XV,1–10.
b) lunar eclipses: VI; X; XI
c) conjunctions of stars and planets: XIV,33–38
d) longitudinal differences (of cities): III,13–15; XV,21–23; 25

e) prime vertical: IV,32–33; 36; 38
f) rising of the moon: V
g) magical diagrams: XIV,27–28; 41
h) graphical constructions: XIV,1–18; 41
i) gnomon shadow: II,9–13; III,10; IV,19–22; 35; 37–38; 41–56; XIII,11; 30–33; XIV,5; 10; 14–15
j) sine of terrestrial latitude: IV,20–21; IV,27–28; 40; XIV,9–10; 18
k) sine of colatitude: IV,23; 28; 40; XIV,8; 18
l) declination: IV,16–18; 24; XIII,10.

It is curious that this list of contents does not mention the rules for calculating planetary positions as given in XVI and XVII.

I,8–15. In these verses Varāhamihira summarizes the rules for computing the ahargaṇa (i.e. the number of days elapsed since epoch) in the three texts associated with Lāṭadeva, i.e., the Romakasiddhānta (I,8–10 and 15), the Pauliśasiddhānta (I,11–13), and the Sūryasiddhānta (I,14).

I,8. The epoch of Lāṭadeva's Romaka is the first tithi after conjunction in the month Caitra at the beginning of the year Śaka 428, i.e., the time of conjunction of Sun and Moon at approximately Aries $0°$. This date is A.D. 505 March 22, Tuesday; see Part I p. 8 in the introduction. The time is sunset at Yavanapura (cf. XV,18), i.e., about 6 P.M. of March 21, local time at Yavanapura. According to III,13 the time difference between Yavanapura and Avantī is $7^1/_3$ nāḍīs or 2 hours 56 minutes; therefore the local time at Avantī is ca. 8;56 P.M. Cf., however, VIII,5 and XV,18.

I,9–10. The purpose of this section is the determination of the number D of days elapsed since epoch at a moment characterized as N completed years of the Śaka era, plus m completed (mean) synodic months, plus τ completed tithis.

This problem is solved by computing first two terms A and B:

$$A = (12(N-427)+m)\frac{7}{228} \tag{1}$$

$$B = (A+12(N-427)+m)\ 30+\tau. \tag{2}$$

In these expressions the factor $12(N-427)+m$ represents the number of lunar months completed since epoch (Ś.E. 428) under the preliminary assumption that only 12 months correspond to each year. According to the "Metonic" 19-year cycle, however, 7 intercalary months must be added to each set of $19 \cdot 12 = 228$ months. Consequently the term A gives the number of intercalary months since epoch, B the total of tithis since epoch.

It is furthermore assumed that

$$703^{\tau} = 692^{\mathrm{d}} = (703-11)^{\mathrm{d}}. \tag{3}$$

The number C of days corresponding to B tithis is therefore

$$C = B - \frac{11}{703} B. \tag{4}$$

From this amount is subtracted a constant

$$c = \frac{514}{703} \tag{5}$$

which leads to the final result

$$D = C - c = B - \frac{11B + 514}{703} = B - \frac{11\,B + 8{,}34}{11{,}43} \tag{6}$$

for the number of days since epoch.

Measured in sexagesimal fractions of a day the constant c corresponds to

$$c \approx 0;43,52,\ldots{}^{\mathrm{d}}. \tag{7}$$

Since $C = D + c$ this means that the interval C begins about $^3/_4$ of one day before epoch. The latter falling at sunset (cf. I,8) C begins at the preceding midnight. The time of epoch being the vernal equinox one should expect $c = {}^3/_4{}^{\mathrm{d}} = 0;45^{\mathrm{d}}$ exactly instead of (7). This would have been obtained by replacing 514 in the numerator of (5) by 527. We cannot explain this small discrepancy.

It follows from (3) that

$$1 \text{ tithi} = 0;59,3,40,11,56,\ldots \text{ days} \tag{8}$$

hence

$$1 \text{ lunar month} = 29;31,50,5,58,\ldots \text{ days} \tag{9}$$

and from the 19-year cycle

$$1 \text{ year} = 365;14,48,4,\ldots \text{ days} \tag{10}$$

Note that in I,15 a tropical year contains 365;14,48 days exactly.

I,11–13. The text gives the following rule for the determination of the number m_{a} of intercalary months:

$$m_{\mathrm{a}} = \frac{10 d_{\mathrm{s}} + 698}{9761} + \frac{y}{300 \cdot 107} + \frac{y}{5506} \tag{1}$$

where d_{s} means the number of saura days contained in the interval under consideration, y the number of complete years in the same interval of time. The term $698/9761 = 11{,}38/2{,}42{,}41 \approx 0;4{,}17$ is obviously an epoch correction which must be due to the fact that an integer number of intercalary months plus this fraction had elapsed between the original epoch of the Pauliśasiddhānta and the epoch of Lāṭadeva.

If we wish to apply (1) for exactly one year we have to substitute $d_s = 360$ and $y = 1$. Computing sexagesimally this gives

$$m_a(1) = \frac{1,0,0}{2,42,41} + \frac{1}{8,55,0} + \frac{1}{1,31,46}$$
$$\approx 0;22,7,43,58,7 + 0;0,0,6,43,44 + 0;0,0,39,13,48$$
$$\approx 0;22,8,29,54.$$

Hence one year contains

$$1^{\mathrm{y}} = 12;22,8,29,54^{\mathrm{m}} \tag{2}$$

synodic months. From III,1 we know that one sidereal year is exactly defined by

$$1^{\mathrm{y}} = 6,5;15,30^{\mathrm{d}} \tag{3}$$

hence from (2)

$$1^{\mathrm{m}} \approx 29;31,48,16,37^{\mathrm{d}}. \tag{4}$$

For the number u of omitted tithis the text gives the rule

$$\left.\begin{aligned} u &= \left(\frac{d}{63} + \frac{\tau}{25\,135}\right)\left(1 - \frac{1}{203\,279}\right) \\ &= \left(\frac{d}{1,3} + \frac{\tau}{6,58,55}\right)\left(1 - \frac{1}{56,27,59}\right) \approx \left(\frac{d}{1,3} + \frac{\tau}{6,58,55}\right)\cdot 0;59,59,58,56 \end{aligned}\right\} \tag{5}$$

d being the number of calendar days, τ the number of tithis contained in the interval under consideration. If we again apply this rule to 1 year we must for d use (3) and for τ, because of (2),

$$1^{\mathrm{y}} = 6,11;4,14,57^{\tau}. \tag{6}$$

Hence for one year

$$u = \left(\frac{6,5;15,30}{1,3} + \frac{6,11;4,14,57}{6,58,55}\right)\cdot 0;59,59,58,56$$
$$\approx (5;47,51,54,17 + 0;0,53,8,50)\cdot 0;59,59,58,56$$
$$\approx 5;48,44,56,55$$

as the number of omitted tithis.

This result can also be obtained directly from (6) and (3) because

$$6,11;4,14,57 - 6,5;15,30 = 5;48,44,57 \tag{7}$$

again represents the number of omitted tithis.

Also from (2) and (3) can be derived the sidereal mean motion of the moon. One finds $13;10,35,37,\ldots^{\circ/\mathrm{d}}$.

I,14. Since 180 000^{y} = 50,0,0^{y} contain 66 389 = 18,26,29 intercalary months, hence 50,0,0 · 12 + 18,26,29 = 10,18,26,29 lunar months, one finds that

$$1^{y} = 12;22,7,46,48^{m} = 6,11;3,53,24^{\tau}. \qquad (1)$$

Since 50,0,0^{y} contains 1 045 095 = 4,50,18,15 omitted tithis their number per year is 5;48,21,54 and therefore with (1)

$$1^{y} = (6,11;3,53,24 - 5;48,21,54)^{d} = 6,5;15,31,30^{d} \qquad (2)$$

as the exact length of the sidereal year; cf. also IX,1.

It follows from (1) and (2) that

$$1^{m} \approx 29;31,50,6,52,59,\ldots^{d} \qquad (3)$$

and

$$\bar{v} \approx 13;10,34,52,6,\ldots^{o/d} \qquad (4)$$

for the sidereal mean motion of the moon.

Lāṭadeva's Sūryasiddhānta, as we know from chapter XVI, employed a Mahāyuga of 4 320 000 = 20,0,0,0 = 24 · 50,0,0 years. If we multiply the parameters in this verse by 24 we find that there are

53 433 336	=	4,7,22,35,36 lunar months
1 593 336	=	7,22,35,36 intercalary months
1 603 000 080	=	2,3,41,17,48,0 tithis
1 577 917 800	=	2,1,45,10,30,0 sāvana days

in one Mahāyuga; these are precisely the parameters of the ārdharātrika (i.e., midnight) system.[1])

I,15. Here it is assumed that 2850 = 47,30 = 2,30 · 19 years contain 1050 = 17,30 = 2,30 · 7 intercalary months. This is the well known scheme of the "Metonic cycle", consistently used in the Babylonian ephemerides. The number of omitted tithis is 16 547 = 4,35,47, therefore the number of days in one year

$$\left(12 + \frac{7}{19}\right)30 - \frac{4,35,47}{2,30 \cdot 19} = \frac{4,49,9,13}{19 \cdot 2,30} = \frac{15,13,7}{2,30} = 6,5;14,48^{d}. \qquad (1)$$

This is exactly the length of the tropical year according to the Hipparchian-Ptolemaic theory (Almagest III,1 p. 208,12 Heiberg). The same value appears again in VIII,1.

The relation (1) can also be formulated as

$$47,30^{y} = 2,30 \cdot 19^{y} = (47,30 \cdot 12 + 17,30)^{m} = 9,47,30^{m} = 2,30 \cdot 3,55^{m} \qquad (2a)$$

i.e.

$$19^{y} = 3,55 \text{ synodic months.} \qquad (2b)$$

[1]) The parameters of the ārdharātrika system are found in Mahābhāskarīya VII,21–35, and in Khaṇḍakhādyaka I, II, and VI. Table 1 shows the number N of sidereal revolutions in a Mahāyuga which consists of 4 320 000 = 20,0,0,0 sidereal years = 1 577 917 800 = 2,1,45,10,30,0 days. For additional planetary parameters see the commentary on XVI,12–14, Table 22.

TABLE 1.

	N
Sun	4 320 000 = 20, 0, 0, 0
Moon	57 753 336 = 4,27,22,35,36
Lunar apogee	488 219 = 2,15,36,59
Lunar nodes	232 226 = 1, 4,30,26
Saturn	146 564 = 40,42,44
Jupiter	364 220 = 1,41,10,20
Mars	2 296 824 = 10,38, 0,24
Venus, śīghra	7 022 388 = 32,30,39,48
Mercury, śīghra	17 937 000 = 1,23, 2,30, 0

Furthermore

$$47{,}30^{y} = 9{,}47{,}30^{m} = 4{,}53{,}45{,}0^{\tau} = (4{,}53{,}45{,}0-4{,}35{,}47)^{d} = 4{,}49{,}9{,}13^{d}. \qquad (3)$$

In VIII,1 this same number of days is equated to 10,35,0 sidereal months:

$$4{,}49{,}9{,}13^{d} = 10{,}35{,}0 = 2{,}30\cdot 4{,}14 \text{ sidereal months} \qquad (4a)$$

hence with (2a)

$$19^{y} = 4{,}14 \text{ sidereal months} \qquad (4b)$$

and finally

$$3{,}55 = 235 \text{ synodic months} = 4{,}14 = 254 \text{ sidereal months} \qquad (5)$$

as basic relation between synodic and sidereal revolutions of the moon, well known in Babylonian astronomy.

Because $7:19 = 0;22{,}6{,}18{,}56{,}50,\ldots$ one finds for the Metonic cycle that

$$1^{y} = 12;22{,}6{,}18{,}56{,}50,\ldots^{m} \qquad (6)$$

and thus with (1)

$$1^{m} = 29;31{,}50{,}5{,}37,\ldots^{d}. \qquad (7)$$

Finally the mean motion of the moon is

$$\bar{v} = 13;10{,}34{,}59{,}50,\ldots^{\circ/d}. \qquad (8)$$

I,16. Summary of the general rules on which the relations are based which were utilized in the preceding verses. For the term "solar measure" cf. pt. I, p. 185.

I,17–21. Let

$$a' = a + 2227 = a + 37{,}7 \qquad (1)$$

be the (augmented) ahargaṇa where a represents the number of days since epoch. Then the following set of rules is given in the text:

$$a' = q \cdot 2520 + r \qquad 0 \leqq r < 2520 = 7 \cdot 360 = 42{,}0 \tag{2}$$

$$r = c_1 \cdot 360 + c_2 \qquad \text{thus } 0 \leqq c_1 < 7, \qquad 0 \leqq c_2 < 360 \tag{3}$$

where c_1 represents the number of completed years. From it

$$(c_1 + 1) \cdot 3 - 2 = c_3 \cdot 7 + c_4 \qquad 0 \leqq c_4 < 7 \tag{4}$$

gives the "lord of the year" c_4. Furthermore

$$\left(\frac{a'}{30} + 1\right) \cdot 2 = c_5 \cdot 7 + c_6 \qquad 0 \leqq c_6 < 7 \tag{5}$$

where c_6 is the "lord of the month", counting from Sunday.[1]) Then

$$a' = c_7 \cdot 7 + c_8 \qquad 0 \leqq c_8 < 7 \tag{6}$$

gives the "lord of the day" c_8 and

$$(c_8 \cdot 3 - 1 + h) \cdot 5 = c_9 \cdot 7 + c_{10} \qquad 0 \leqq c_{10} < 7 \tag{7}$$

the "lord of the h-th hour".

Let

$$\text{A B C D E F G} \tag{8a}$$

be the order of days in the week, ruled by the planets

$$\odot \quad ☾ \quad ♂ \quad ☿ \quad ♃ \quad ♀ \quad ♄ \tag{8b}$$

respectively. Then the rule (7) results for the first hour ($h = 1$) in

$$c_8 \cdot 15 = 2c_8 \cdot 7 + c_8$$

hence in

$$c_{10} = c_8$$

i.e. the lord of the first hour is also the lord of the day.

For $h = 25$ one obtains

$$c_8 \cdot 15 + 120 = (2c_8 + 17)7 + c_8 + 1$$

i.e. the first hour of the next day has the lord $c_8 + 1$ as it should be. For $h = 2$ one has

$$c_8 \cdot 15 + 5 = 2c_8 \cdot 7 + c_8 + 5.$$

This means: if we begin for $h = 1$ with A then $h = 2$ has the lord F, $h = 3$ the lord D, etc. In this way one obtains for consecutive hours the following order of the planets

$$\odot \quad ♀ \quad ☿ \quad ☾ \quad ♄ \quad ♃ \quad ♂ \quad \text{etc.}$$

[1]) The rule given in (5) is wrong. The text's (I,19) "increase the (resulting) months by the current one" should be replaced by "discard the fractional part of the current (month)".

i.e. the Greek order of the planets from which the order (8b) for the days of the week originated.

From (5) it follows that an increase of a' (or of a) by 30 increases the lord of the month by 2, as it should be since $30 \equiv 2$ mod. 7. Of course "month" means here a fixed interval of 30 days and not a lunar month.

For the first day of the ahargaṇa one has $a' = 2227 + 1$ hence from (6)

$$2228 \equiv 318 \cdot 7 + 2$$

hence $c_6 = 2$ i.e. Tuesday.

The rule (4) can also be written as

$$3c_1 \equiv c_4 - 1 \text{ mod. } 7.$$

Each unit of c_1 represents according to (2) a schematic "year" of 360 days. Hence c_4 increases by 3 if c_1 increases by 1 as is to be expected since $360 \equiv 3$ mod. 7. For $a' = 1$ one has $c_1 = 0$ thus $c_4 = 1$ i.e. Sunday.

I,22. Varāhamihira explains the astrological influences of the lords of the year, the lords of the month, and the lords of the day in Bṛhatsaṃhitā XIX.

I,23–25. Varāhamihira here discusses the Persian year of the Maga Brāhmaṇas.

I,23. If we now consider a "year" as containing 365 days, the quotient

$$a + 1 \equiv c_5 \text{ mod. } 365 \qquad 0 \leqq c_5 < 365$$

gives the fraction c_5 of one year elapsed at the given moment a. Furthermore

$$c_5 \equiv c_6 \text{ mod. } 30 \qquad 0 \leqq c_6 < 30$$

refers to the number of days within the current schematic month of 30 days. This last residue c_6 defines the "lord of the degree".

I,24–25. Here are listed the Sanskrit "equivalents" of the thirty angels who rule the days of a Persian month. Column I below gives their names according to the Bundahishn (cf. A. Christensen, L'Iran sous les Sassanides, 2nd ed. Copenhague 1944 p. 158), and column II Varāhamihira's list.

	I	II
1.	Ōhrmazd	Kamalodbhava = Brahman
2.	Vahman = "Good Thought"	Prajeśa = Prajāpati
3.	Urdvahisht = "Best Truth"	Svargeśa = "Lord of Heaven"
4.	Shahrēvar = "Desirable Domination"	Śāstṛ = "Ruler"

5.	Spandarmadh = "Spiritual Purity"	Rudra
6.	Khvardādh = "Integrity"	Manyu = "Mind"
7.	Amurdādh = "Immortality"	Vasu = "Wealth"
8.	Dadhv = "Creator"	Kamalā = Lakṣmī
9.	Ādhur = "Fire"	Anala = "Fire"
10.	Ābhān = "Water"	Antara = "Death"
11.	Khvar = "Sun"	Vayaḥ = "Sun"
12.	Māh = "Moon"	Śaśi = "Moon"
13.	Tīr = "Mercury"	Indra
14.	Gōsh = "Bull"	Go = "Bull"
15.	Dadhv = "Creator"	Nirṛti = "Destruction"
16.	Mihr = "Sun"	Hara = Śiva
17.	Srōsh = "Obedience"	Bhava = "Being"
18.	Rashn = "Truth"	Guru = "Teacher"
19.	Fravardīn = "Genii"	The Pitṛs = "The Fathers"
20.	Varhrān	Varuṇa
21.	Rām = "Joy"	Baladeva
22.	Vādh = "Wind"	Samīraṇa = "Wind"
23.	Dadhv = "Creator"	Yama = "Death"
24.	Dēn = "Religion"	Vāk = "Word"
25.	Ard = "Retribution"	Śrī = "Prosperity"
26.	Ashtādh = "Rectitude"	Dhanada = Kubera
27.	Asmān = "Sky"	The Giris = "The Mountains"
28.	Zāmdādh = "Earth"	Dhātrī = "Earth"
29.	Mahrspand = "God's Word"	Vedhāḥ = "Pious"
30.	Anaghrān = "Infinite Lights"	Paraḥ Puruṣa = Viṣṇu

The angels of lists I and II are reasonably similar for days 1, 2, 3, 4, 5(?), 7, 9, 11, 12, 14, 18(?), 19, 22, 24, 28, and 29, and perhaps for some others. The variations may be due in part to the fact that Varāhamihira's is a Maga Brāhmaṇa list, the Bundahishn's probably a Zurvanite document.

Chapter II

II,1. The rule of the text implies the use of a julian year since

$$\frac{a \cdot 4 + 6}{1461} = \frac{a + 1;30}{365;15}. \tag{1}$$

The division of the ahargaṇa a by $365;15^{d}$ will leave a remainder which gives the number of quarter-days after the vernal equinox. The twelve coefficients enumerated at the end of the verse, $126 - 1 = 125$, $126 - 0 = 126$, etc., give the number of quarter

Table 2.

	d/4	d	Season	Velocity
♈	125	31;15		0;57,16,48°/d
♉	126	31;30		0;57, 8,34,...
♊	126	31;30	94;15	0;57, 8,34,...
♋	126	31;30		0;57, 8,34,...
♌	124	31; 0		0;58, 3,52,...
♍	122	30;30	93; 0	0;59, 0,59,...
♎	119	29;45		1; 0,30,15,...
♏	117	29;15		1; 1,32,18,...
♐	117	29;15	88;15	1; 1,32,18,...
♑	118	29;30		1; 1, 1, 1,...
♒	120	30; 0		1
♓	121	30;15	89;45	0;59,30,14,...
Total	1461	365;15		

days for the travel of the Sun in the consecutive zodiacal signs. This leads for the solar motion to the pattern shown in Table 2.

The additive constant (kṣepa) 6 in the numerator (i.e. $^6/_4 = 1^1/_2$ days) indicates that the author supposed the vernal equinox to occur not at the epoch, but $1^1/_2$ days earlier.

The graph for the corresponding velocities (cf. Fig. 1) shows that the values for Pisces and Aries cannot be correct. A simple emendation would be 124 for Aries and correspondingly 122 for Pisces. The text, however, does not permit such a correction.

II,2–6. We have here rules for the determination of the longitude of the Moon, rules which are of great historical interest because their Hellenistic, ultimately Babylonian, prototypes are known.[1])

Let a be the ahargaṇa, then one takes as number of days

$$n = a + 1936 = a + 32{,}16 \qquad (1)$$

i.e. one introduces a point of departure about $5^1/_3$ years back from the accepted epoch date. We shall return to this modified epoch date later and show that it represents a position of the Moon at its apogee (cf. p. 22).

From n one derives numbers α and β:

$$n = \alpha \cdot 3031 + \beta \qquad 0 \leqq \beta < 3031 \qquad (2)$$

where α (called "ghana") counts the number of periods of 3031 = 50,31 days length. This number 3031 represents the length of 110 anomalistic months as we know, e.g., from VIII,5.

[1]) Cf., e.g., the Greek papyri P. Lund 35a and P. Ryl. 27 (cf. below p. 152, Bibliography 1,D) and ACT I passim.

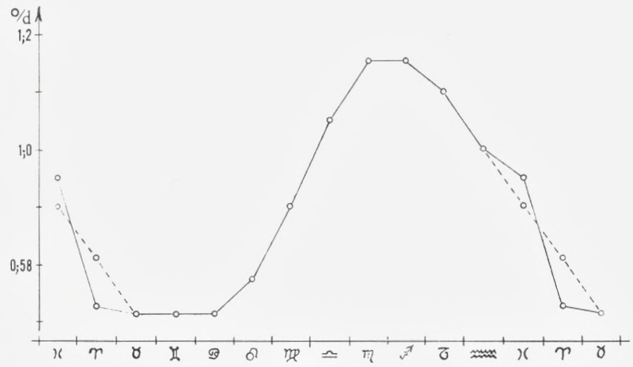

Fig. 1.

With the remainder β one forms

$$\frac{9\beta}{248} = m + \frac{9t}{248} \qquad 0 \leqq t < \frac{248}{9}. \tag{3}$$

Since

$$\tfrac{248}{9} = 27;33,20^{\mathrm{d}} \tag{4}$$

is the length of one anomalistic month, one can replace (3) by

$$\beta = m \cdot 27;33,20 + t \qquad 0 \leqq t < 27;33,20 \tag{5}$$

where m (called "gatis") gives the number of anomalistic months contained in β. The text counts the residue t in "padas" where (cf. (4))

$$1 \text{ pada} = \tfrac{1}{248} \text{ anomal. month} = \tfrac{1}{9} \text{day}. \tag{6}$$

Having determined the integers α, m, and t the corresponding increments of longitude λ_1, λ_2, λ_3 respectively will be computed.

First, with α from (2):

$$\alpha = 16u + v \qquad 0 \leqq v < 16 \tag{7}$$

and from it

$$\lambda_1 = 69;7,1^\circ - \tfrac{3}{4}v \cdot 30^\circ + 0;2\alpha. \tag{8}$$

This means that for $n = 0$ the Moon had the longitude

$$\lambda_0 = 69;7,1^\circ. \tag{9}$$

For this parameter cf. below p. 22. Otherwise (7) and (8) are based on the assumption that the longitude of the Moon increases during one ghana by $337\tfrac{1}{2}^\circ + 0;2^\circ = 5,37;32^\circ$ — corresponding to a mean motion of $13;10,34,52,46,\ldots^{\circ/\mathrm{d}}$.

Indeed, since

$$5,37;30 \equiv -22;30 = -\tfrac{3}{4}30 \qquad \text{mod. } 6,0$$

one has for α ghanas, with (7):

$$-\alpha\tfrac{3}{4}30 = -(16u+v)\tfrac{3}{4}30 = -12\cdot 30u - \tfrac{3}{4}v\cdot 30 \equiv -\tfrac{3}{4}v\cdot 30 \qquad \text{mod. } 6{,}0$$

which explains the increment depending on v in (8).

The contribution to the longitude during m gatis is assumed to be

$$\lambda_2 = m(185 - \tfrac{1}{6}) \text{ minutes} = m\cdot 3;4{,}50^\circ. \tag{10}$$

That is to say the Moon moves in one anomalistic month 6,3;4,50°, or, using (4), the Moon's mean velocity is taken to be 13;10,34,43,3,...$^{o/d}$. This is slightly less than the value obtained from (7) and (8).[1])

The increments λ_1 and λ_2 take care of complete anomalistic periods of the Moon. What remains is only the fraction t in (5) of the current anomalistic period. For this last contribution a definite velocity pattern is assumed, beginning with minimum velocity. This implies that the Moon had minimum velocity at $n = 0$.

In order to compute the increase in longitude of the Moon during a fraction of an anomalistic month one has to distinguish between the two halves of a month. During the first half the motion is accelerating, during the second half the velocity decreases again toward its minimum. The two halves are characterized in the text by the number of "padas" (cf. (6)) within the month: the first half of 124 padas contributes (according to II,5) 180;4° of longitudinal progress, the second half should complete the anomalistic period. According to (10) one should expect a total of about 363° in longitudinal gain but for the sake of greater computational convenience the motion during the last part of the month is slightly modified.

In describing the rules of the text we denote the longitudinal increment during the first half of the month by λ^+, during the second half by λ^-. Reckoning the residue t found in (3) or (5) as p "padas" the rules of the text are

$$\left.\begin{array}{c}\text{first half: } \lambda^+ = p^\circ + (1094 + 5(p-1))\dfrac{p}{63} \text{ minutes} \\ 0 \leqq p \leqq 124\end{array}\right\} \tag{11}$$

$$\left.\begin{array}{c}\text{second half: } \lambda^- = (p-124)^\circ + (2414 - 5(p-124-1))\dfrac{p}{63} \\ +\ \lambda^+(124) \text{ minutes} \qquad 125 \leqq p \leqq 248\end{array}\right\} \tag{12}$$

where $\lambda^+(124)$ represents the value obtained from (11) if one substitutes $p = 124$, or, as said in the text

$$\lambda^+(124) \approx 180;4^\circ. \tag{12a}$$

[1]) If one were to emend in (10) the 1/6 to 0;6 = 1/10 one would have a motion of 3;4,54° (+360°) in one anomalistic month and hence a mean motion of 13;10,34,52,...$^{o/d}$, i.e., practically the same value as in (7) and (8). On the other hand, the parameters in P. Ryl. 27 lead to 3;4,49,26,...° as progress per anomalistic month.

Indeed, for $p = 124$ one obtains from (11) the value $124^\circ + (3363 + \frac{47}{63})$ minutes $= 180^\circ + (3 + \frac{47}{63})$ minutes $\approx 180;4^\circ$.

In order to discuss the astronomical signifcance of these rules it is convenient to reconvert the padas to days t and to write the numbers sexagesimally with degrees as units.

For the accelerating, i.e. first, half of the month one can replace (11) by

$$\lambda^+ = 11;42\,t + \frac{0;45}{7}\,t(t-1) = mt + \frac{d}{2}\,t(t-1). \tag{13}$$

The velocity (in degrees per day) which produces such a motion is found by forming the differences

$$v^+ = \Delta\lambda^+ = \lambda^+(t+1) - \lambda^+(t) = m + dt. \tag{14}$$

Hence v^+ is a linearly increasing function of time with the minimum

$$m = 11;42^{\circ/\mathrm{d}} \tag{15a}$$

and the difference

$$d = \frac{1;30}{7}. \tag{15b}$$

For the second half of the month we count the days t' beginning with the mid-point $p = 124$, i.e. we define

$$t' = \tfrac{1}{9}(p - 124). \tag{16}$$

Then (12) transforms itself into

$$\lambda^- = 3,0;3 + \frac{0;47}{1,3} + \left(14;39 + \frac{1}{7,0}\right)t' - \frac{0;45}{7}\,t'(t'-1) = c + Mt' - \frac{d}{2}t'(t'-1) \tag{17}$$

with

$$c = 3,0;3 + \frac{0;47}{1,3} \approx 3,0;4^\circ \tag{18a}$$

$$M = 14;39 + \frac{1}{7,0} = 14;39,8,34,17,\ldots{}^{\circ/\mathrm{d}} \tag{18b}$$

and $d = \dfrac{1;30}{7}$ as before in (15b). The velocity in the second half is therefore given by

$$v^- = \Delta\lambda^- = M - dt'. \tag{19}$$

The fact that d has the same value in (14) and (19) shows that the velocity underlying (11) and (12) is a linear zigzag function with extrema given by (15a) and (18b) and difference (15b). Consequently we find for the period of this function

$$P = \frac{2\,(M - m)}{d} = \frac{20,40}{7} \cdot \frac{7}{45} = \frac{248}{9} = 27;33,20^{\mathrm{d}} \tag{20}$$

as it should be according to (4). For the mean value one finds

$$\mu = \tfrac{1}{2}(m+M) = 13;10 + \frac{4}{7,0} \approx 13;10,34,17,\ldots{}^{o/d}. \tag{21}$$

Essentially the same linear zigzag function for the lunar velocity is described in III,4, the only difference being that the value (18b) for M is rounded to $14;39^{o/d}$ which would mean $\mu = 13;10,30^{o/d}$ instead of (21).

We must still explain the value (18a) of the constant c in (17). Obviously the longitudinal gain over one complete anomalistic period should be μP. Hence, one should have at the end, i.e. for $t' = \frac{P}{2}$ from (17):

$$\mu P = c + M \cdot \frac{P}{2} - \frac{d}{2} \cdot \frac{P}{2} \cdot \left(\frac{P}{2} - 1\right) \tag{22}$$

or

$$\mu = \frac{c}{P} + \frac{M}{2} - \frac{d}{4}\left(\frac{P}{2} - 1\right)$$

$$= \frac{9}{4,8}\left(3,0;3 + \frac{47}{1,3}\right) + \tfrac{1}{2}(14,39 + \tfrac{1}{7}) - \frac{45}{14} \cdot \frac{1,55}{9} = 13,11$$

exactly. This shows that c was computed from the necessary relation (22) by using for the mean motion the rounded value

$$\mu \approx 13;11^{o/d}. \tag{23}$$

It is of interest to investigate the lunar equation which results from the pattern of the true longitude λ^+, computed according to the above scheme. For this end one has to compute λ^+ from (13) for $t = 1, 2, \ldots$ and similarly the corresponding mean positions $\bar{\lambda}$ from day to day, beginning with the apogee, and to form the differences

$$\theta = \lambda^+ - \bar{\lambda}. \tag{24}$$

Table 3 and its graphical representation in Fig. 2 show the result. From it it is clear that the maximum equation is

$$\theta_{\max} = 5;5^\circ \tag{25}$$

which occurs at $t = 7$, i.e., as expected, near $t = \frac{P}{4} = 6;53,20$.

It should be emphasized that this is a necessary consequence of the velocity function determined by (14) and (19). In other words (25) is not a parameter which can be chosen freely after v has been fixed; cf. below the discussion to III,4 and III,5–8.

The origin of the other parameters can only be reconstructed with a fair degree of plausibility. In the construction of a linear zigzag function the only parameter

TABLE 3.

t	$\bar{\lambda}$	λ^+	$-\theta$	
1	784.14	702.00	82.1′ =	1;22°
2	1568.28	1416.86	152.4	2;32
3	2352.42	2144.57	207.8	3;28
4	3136.57	2885.14	251.4	4;11
5	3920.71	3638.57	282.1	4;42
6	4704.85	4404.86	300.0	5; 0
7	5489.00	5184.00	305.0	5; 5
8	6273.14	5976.00	297.1	4;57
9	7057.28	6780.86	276.3	4;36
10	7841.42	7598.57	242.8	4; 3
11	8625.57	8429.14	196.4	3;16
12	9409.71	9272.57	137.1	2;17
13	10193.85	10128.86	65.0	1; 5
13;47	10803.74	10803.75	0.0	0

which must be strictly preserved is the period P, here represented by the classical Babylonian parameter (20). For the convenience of actual computation it is essential that d be a small number. A crude estimate of the anomalistic lunar motion, $P \approx 28^d$, $M - m \approx 3°$ would give for d

$$d = \frac{M-m}{P} = \frac{1;30}{7}$$

as in (15b). Again a crude estimate for the velocities would be $\mu \approx 13;11$, hence $m \approx 11;41$, $M \approx 14;41$. Starting with these estimates one must improve μ by coming nearer to the well known mean value 13;10,35. If one wishes to preserve d which contains the fraction 1/7 it is convenient to take also for μ the nearest approximation with this fraction, i.e. $\mu = 13{,}10 + \frac{4}{7}$ which is (21). Finally, one must use the accurate value (20) for P and this, with (21), leads to (15a) and (18b) for the extrema. At any rate, arithmetical considerations more or less following the here described lines

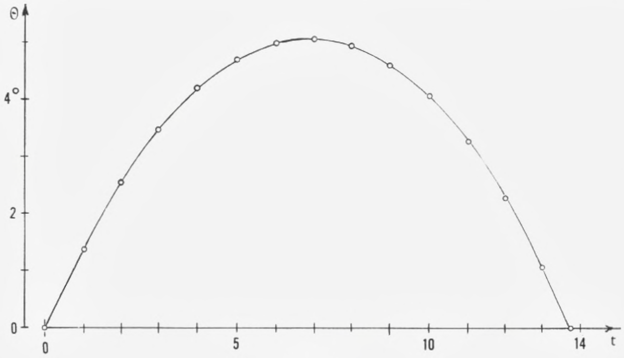

Fig. 2.

must lie at the basis of the rules given in the text and certainly not any detailed observations beyond the well known Babylonia parameters.

As a result of the prescribed operations one has found the contributions to the motion in longitude λ_1, λ_2, and $\lambda_3 = \lambda^+$ or λ^- respectively. Hence one obtains for the longitude λ of the Moon at the moment t

$$\lambda = \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 \tag{26}$$

where λ_0 is the lunar longitude 1936 days before epoch, i.e. before 505 March 22 (cf. above p. 16 (1)), given in II,3:

$$\lambda_0 = \text{♊}\ 9;7,1. \tag{27}$$

The correctness of this element can be easily checked with modern tables. Computing for 499 Dec. 3, 5 P.M. (a) for the meridian of Ujjayinī (b) for the meridian of Alexandria, one finds[1])

$$\text{Ujjayinī:}\ \lambda = 69.33 = \text{♊}\ 9;20 \qquad \text{anomaly } 186^\circ$$

$$\text{Alexandria:}\ \lambda = 70.82 = \text{♊}\ 10;49 \qquad \text{anomaly } 187;30^\circ$$

as compared with $\lambda_0 = \text{♊}\ 9;7$ and anomaly 180° assumed in the text. The computed data obviously confirm in principle the data of the text but do not provide a clear enough distinction between Ujjayinī and Alexandria.

It is not clear why just this particular apogee position had been chosen instead of the nearest one to the epoch A.D. 505 of the ahargaṇa, unless the Vasiṣṭhasiddhānta was revised in 499 as the Romaka, Pauliśa (and Sūrya) were revised by Lāṭadeva in 505.

II,7. We adopt here the following notation for longitudinal arcs:

$$\text{s...sign:} \qquad 1^{s} = 30^\circ$$

$$\text{na...nakṣatra:} \qquad 1^{na} = 13;20^\circ = \frac{4^{s}}{9}$$

$$\mu\text{...muhūrta:} \qquad 1^{\mu} = \frac{1^{na}}{30} = 0;26,40^\circ = \frac{4^\circ}{9}.$$

This explains the rule of the text:

$$\lambda^\circ = a^{s} + b^\circ = \tfrac{4}{9}a^{na} + \tfrac{4}{9}b^{\mu}.$$

Let $\Delta\lambda$ be the elongation of the Moon from the Sun. In one lunar month, which, by definition, contains 30 tithis (τ), the elongation increases by 360°, hence 12° per tithi. Consequently for an elongation of 1^{s} is required the time $\frac{30}{12} = \frac{5^{\tau}}{2}$, hence $\frac{5}{2}\Delta\lambda^{\tau}$ for an elongation of $\Delta\lambda^{s}$. Cf. also III,16.

[1]) Using the tables of P. V. Neugebauer, Tafeln zur astronomischen Chronologie II (1914) and his Chronologie II (1929) Tafel E 1.

II,8. One muhūrta is $^1/_{30}$ of a nychthemeron, i.e. $0;48^h$. When the Sun is at the beginning of Capricorn the length of daylight is assumed to be $9+3 = 12$ muhūrtas, i.e. $9;36^h$. For each subsequent sign 1 muhūrta is added until Aries. From there on the number of signs has to be added to 15 until Cancer and similarly for the length of night which increases after Cancer. The result is a linear zigzag function for the length of daylight with a ratio 2:3 for shortest to longest daylight (cf. Table 4). For this originally Babylonian ratio see Sphujidhvaja's Yavanajātaka 1,68 and 79,31; cf. also Varāhamihira's Bṛhajjātaka I,19, and XII,5 below.

TABLE 4.

0°	Daylight		0°
♑	12^{muh} =	$9;36^h$	♑
♒	13	10;24	♐
♓	14	11;12	♏
♈	15	12	♎
♉	16	12;48	♍
♊	17	13;36	♌
♋	18	14;24	♋

II,9–10. Rules for the length s_n of the noon shadow of a vertical gnomon at a locality for which the geographical latitude φ equals the obliquity ϵ of the ecliptic. Consequently $s_n = 0$ at the summer solstice, i.e. at a solar longitude $\lambda = 3^s$. It is furthermore assumed that s_n increases linearly with 2 units per sign:

$$\left.\begin{aligned} s_n &= 2a && \text{for } \lambda = (3+a)^s \\ s_n &= 12-2a && \text{for } \lambda = (9+a)^s \end{aligned}\right\} \quad a = 0, 1, \ldots, 6$$

Conversely one can find the solar longitude from the length s_n of the noon shadow:

$$\lambda = \left(\frac{s_n}{2} + 3\right)^s \quad \text{for} \quad ♋0 \leqq \lambda \leqq ♑0$$

$$\lambda = \left(15 - \frac{s_n}{2}\right)^s \quad\quad ♑0 \leqq \lambda \leqq ♋0.$$

For the equinoctial noon shadow ($a = 3$) and with $\varphi = \epsilon$ one would have a gnomon

$$g = 6 \cot \epsilon \approx 6 \cdot 2;15 = 13;30.$$

Because of the crudeness of the scheme a gnomon of length 12 is not quite excluded by this estimate.

II,11–13. Let $\lambda(H)$ be the longitude of the rising point of the ecliptic, λ the longitude of the Sun, s_n the noon shadow, s the shadow at any time of the day. Then the text states that

$$\left.\begin{aligned} \lambda(\mathrm{H}) &= \lambda + \frac{36}{12+s-s_{\mathrm{n}}} && \text{before noon} \\ \lambda(\mathrm{H}) &= \lambda + \left(6 - \frac{36}{12+s-s_{\mathrm{n}}}\right) && \text{after noon} \end{aligned}\right\} \quad (1)$$

and conversely

$$\left.\begin{aligned} s &= s_{\mathrm{n}} - 12 + \frac{64800}{\lambda(\mathrm{H})-\lambda} && \text{before noon} \\ s &= s_{\mathrm{n}} - 12 + \frac{64800}{10800-(\lambda(\mathrm{H})-\lambda)} && \text{after noon} \end{aligned}\right\} \quad (2)$$

where $\lambda(\mathrm{H})-\lambda$ should be reckoned in minutes of arc.

Since (2) results from (1) by solving for s we need only to discuss (1). Substituting in (1) $s = s_{\mathrm{n}} = 0$ one obtains $\lambda(\mathrm{H}) = \lambda + 3$ signs which is correct for $\lambda =$ ♋0° under the assumption that the sun is in the zenith as would be the case for $\varphi = \epsilon$ (cf. II,9–10). Substituting $s = \infty$ one finds $\lambda = \lambda(\mathrm{H})$ or $\lambda = \lambda(\mathrm{H}) + 6$ signs as is correct for sunrise or sunset respectively.

If one generally substitutes $s = s_{\mathrm{n}}$ one obtains $\lambda(\mathrm{H}) = \lambda + 3$ signs which is, however, not generally true because the culminating point of the ecliptic and the nonagesimal need not to coincide.

Chapter III

III,1. The rule that the number of revolutions of the Sun is obtained from the ahargaṇa a by multiplying it by 120/43831 = 2,0/12,10,31 = 1/6,5;15,30 shows that one sidereal year is assumed to be 365;15,30 days long. This value is well known during the Middle Ages. Battānī, e.g., calls it "Babylonian".[1]

III,2–3. The equation of center for the Sun is given for 30° sections of anomaly. Since one is directed to add 20° to the longitude of the Sun it is clear that these sections begin at 20° of each sign. Hence the solar apogee is located at ♊20°, as in the ārdharātrika system.

Fig. 3 shows as a continuous curve the function 1;12 sin α. The points marked by × give the values of the equation found in the text at arguments 10°, 40°, 70°, etc. (cf. Table 5). Since ♊20 corresponds to $\alpha = 0$ we may also say that the points × correspond to ♋0°, ♌0°, . . . etc. The very close agreement of the points × with 1;12 sin α shows that the maximum equation for the Sun was assumed to be 1;12°, almost exactly one half of the value in the Almagest (2;23°).

III,4 and 9. The lunar velocity v is here described as a linear zigzag function with the following parameters

[1]) Cf. Nallino, Battānī I p. 40 and p. 127 (4).

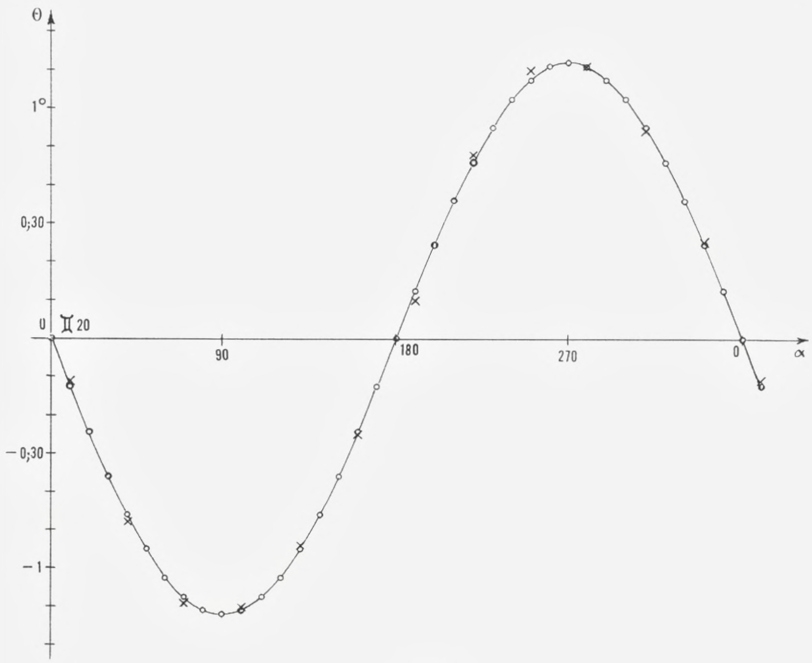

Fig. 3.

$$\left.\begin{aligned} m &= 702 \text{ minutes in } 9 \text{ padas, i.e. } 11;42^{\circ/d} \\ M &= 879 \text{ minutes in } 9 \text{ padas, i.e. } 14;39^{\circ/d} \\ d &= \frac{9 \cdot 10}{7} \text{ in } 9 \text{ padas, i.e. } \frac{1;30^{\circ/d}}{7} \text{ per day.} \end{aligned}\right\} \quad (1)$$

This is the same function which we derived from II,2 to 6, excepting M which had previously the value $14;39 + \frac{1}{7,0}$ as is necessary if one wishes to obtain the exact

TABLE 5.

α		$\lvert 1;12 \sin\alpha \rvert$	Text				α	
0		0					360	
10	170	0;12,30	−0;11		+0;10		190	350
20	160	0;24,39		−0;25		+0;25	200	340
30	150	0;36					210	330
40	140	0;46,17	−0;48		+0;48		220	320
50	130	0;55, 9		−0;54		+0;54	230	310
60	120	1; 2,21					240	300
70	110	1; 7,39	−1; 9		+1;10		250	290
80	100	1;10,54		−1;10		+1;11	260	280
90		1;12					270	

period of 248/9 days for the anomalistic month. Obviously the value of M in (1) is only a convenient rounding for the value in II,2–6.

Separated from III,4 by different rules for the lunar equation (III,5–8) is III,9 where it is said that the longitude of the Moon $\lambda(t+1)$ at the moment $t+1$ is found from $\lambda(t)$ by adding to it $v(t+1)$ as determined from (1).

III,5–8. These verses deal with the determination of the lunar equation, i.e. with the correction θ which must be added to, or subtracted from, the mean longitude to give the true longitude. This function θ has the value zero at the apogee and the perigee of the lunar orbit. Otherwise it is here assumed that the differences $\Delta\theta$ form a linear zigzag function of period $P/2$, $P = 27;33,20^d$ being the anomalistic month. Consequently θ is represented by a difference sequence of second order.

The rules of the text distinguish between the first and the second half of the anomalistic month, beginning at the apogee. As in II,2–6 the anomalistic month is again divided into 248 "padas" (cf. p. 17) such that $1p = \frac{1^d}{9}$.

For the first half, or more specifically, for

$$0 \leqq p \leqq 120 \quad \text{or} \quad 0 \leqq t \leqq 13;20^d \tag{1a}$$

one finds θ (i.e. the negative equation) from

$$\theta = (5261 - 40(p-1))\frac{p}{729} \tag{1b}$$

reckoned in minutes. Proceeding again as in II,2–6 one can replace (1b) by the equivalent rule

$$\theta = 1;1 \cdot t - \left(0;4 + \frac{0;4}{9}\right)t(t-1) \tag{2}$$

which shows that

$$\Delta\theta = 1;1 - \left(0;8 + \frac{0;8}{9}\right)t = 1;1^\circ - 0;8,53,20 \cdot t. \tag{3}$$

The maximum equation should occur at $t = \frac{1}{4}P = 6;53,20^d$. Substituting this value in (2) one obtains

$$\theta_{\max} = 3;59,55,\ldots{}^\circ \approx 4^\circ \tag{4}$$

while (3) gives $\Delta\theta = -0;0,14,\ldots \approx 0$ as it should be. The equation (4) would correspond to an eccentricity of 3;49 for $R = 60$.

In the discussion of II,2–6 we have seen (p. 20) that the velocity function (1) of III,4 leads to a maximum equation of 5;5°. Consequently it is certain that III,5–8 is unrelated to III,4 and III,9 which are the equivalent to II,2–6.

One can again ask how the parameters could have been determined which underly the rules (2) and (3). Obviously one intended to construct a function θ of the type shown in Fig. 2 p. 21 i.e. a sequence of second order such that

$$\Delta\theta = a - dt \tag{5}$$

and therefore

$$\theta = at - \tfrac{1}{2}dt(t-1). \tag{6}$$

Hence our question boils down to the problem of determining the two parameters a and d.

Here one has to start with

$$\theta_{\max} = 4^\circ \quad \text{at} \quad t = \frac{P}{4} = \frac{1{,}2}{9}. \tag{7}$$

At that point $\Delta\theta$ must be zero; thus from (5)

$$a = d \cdot \frac{P}{4}. \tag{8}$$

Substituting this relation in (6) one finds

$$\theta_{\max} = \tfrac{1}{32}\, dP(P+4)$$

hence

$$d = \frac{32}{P(P+4)}\theta_{\max}. \tag{9}$$

Using the values from (7) this gives

$$d = \frac{5{,}24}{36{,}41} = 0;8{,}49{,}56,\ldots \approx 0;8{,}50 \approx 0;8 + \frac{0;8}{9} = \frac{1;20}{9} \tag{10a}$$

$$a = \frac{1;20 \cdot 1{,}2}{1{,}21} = 1;1{,}18,\ldots \approx 1;1. \tag{10b}$$

One would have obtained exactly $d = 1;20/9$ if $P \approx 27;27,30$ and from it exactly $a = 1;1$ if $P \approx 27;27$. The effect of these inaccuracies is clearly visible in Fig. 4.

As a result of these operations one can determine the equation θ which concerns the time between 0 and $\frac{P}{2}$ (and should be subtracted from the mean positions). For the second half of the interval the equation could have been found simply on the basis of symmetry, counting backwards from the endpoint of the anomalistic month. In fact, however, the text instructs us (in III,6) to use for $p > 120$ the amounts

$$p' = p - 120 \tag{11}$$

and to find the equation θ' from the same expression as θ in (1b)

$$\theta' = (5261 - 40(p'-1))\frac{p'}{729} \tag{12}$$

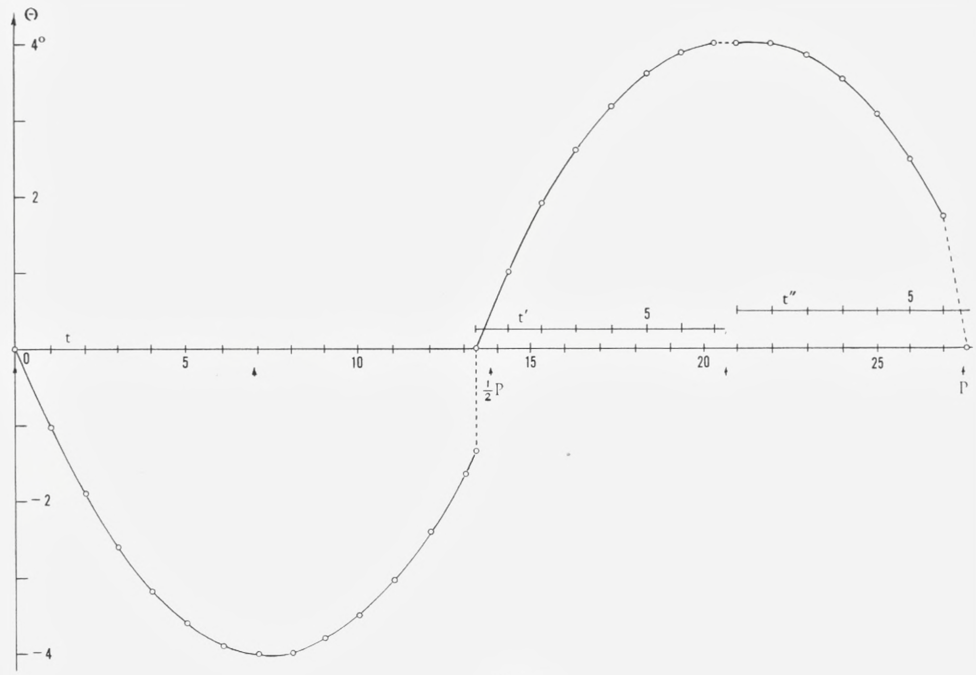

Fig. 4.

but only as long as

$$0 < p' \leqq 63. \tag{13}$$

What happens from here on is difficult to motivate. If we interpret the text correctly (as is by no means certain) the value found for $p' = 63$ is kept valid until $p' = 70$. At $p' = 70$, however, a value is assumed as found from (12) for $p = \frac{560}{9} = 62^2/_9$, i.e. a value which is practically $\theta_{\max}$. But by moving it to $p' = 70$ the maximum area is extended instead of shortened (as θ should have shown; cf. Fig. 4). In other words from $p' = 70$ on one introduces a new variable

$$p'' = p' - 69 \tag{14}$$

and computes

$$\theta' = (5261 - 40(p'' - 1))\frac{p''}{729}. \tag{15}$$

This leads for $p'' = 55$ to an equation of about 1;54° instead of to a value nearly zero. The text seems to say that $\theta = 0$ for $p' = 60$; actually $p' = 59$ would correspond to P. It would have been much better to use (12) for the whole second half of an anomalistic month.

III,9 see III,4.

TABLE 6.

t	θ	t	t′	θ'
1^d	1; 1	$13;20^d$	0	0
2	1;53, 6,40	14;20	1	1; 1
3	2;36,20	15;20	2	1;53, 6,40
4	3;10,40	16;20	3	2;36,20
5	3;36, 6,40	17;20	4	3;10,40
6	3;52,40	18;20	5	3;36, 6,40
7	4; 0,20	19;20	6	3;52,40
8	3;59, 6,40	20;20	7	4; 0,20
9	3;49	t	t″	
10	3;30			
11	3; 2, 6,40	21	0	4; 0, 1,...
12	2;25,20	22	1	3;59,34,...
13	1;39,40	23	2	3;50,13,...
p =		24	3	3;31,59,...
120 13;20	1;22,26,...			
		25	4	3; 4,52,...
14	0;45, 6,40	26	5	2;28,51,...
		27	6	1;43,57,...
		27;40		0

III,10–11. We associate the indices $i = 0, 1, 2, 3$ with the longitudes 0, 30, 60, 90 respectively. The remaining quadrants of the ecliptic need no special discussion since all steps can be repeated with proper symmetries.

The text gives rules for finding the length of daylight C_i counted in vināḍīs where $360° = 3600^{\text{vin}}$. At equinox $C_0 = 1800^{\text{vin}}$ and in general

$$C_i = 1800 + 2\omega_i \tag{1}$$

where ω_i is the arc ω shown in Fig. 5, for $\lambda = i \cdot 30$. The geographical latitude φ is characterized by the equinoctial noon shadow s_0 of a vertical gnomon of length $g = 12$. The text gives coefficients γ_i such that

$$s_0 \gamma_i = 2(\omega_i - \omega_{i-1}). \tag{2}$$

Since obviously $\omega_0 = 0$ the formula (2) suffices to find all ω_i and thus all C_i. The values given for the γ_i are

$$\gamma_1 = 20^{\text{vin}} \quad \gamma_2 = 16;30^{\text{vin}} \quad \gamma_3 = 6;45^{\text{vin}}. \tag{3}$$

It is not difficult to derive, at least approximately, these values from other known data. Let δ_i be the declination of the Sun. Then Fig. 5 shows that

$$\cot\varphi = \frac{\tan\delta}{\sin\omega}. \tag{4}$$

Now

$$\cot\varphi = g/s_0 \tag{5}$$

and

$$\tan\delta = R\sin\delta/r = \operatorname{Sin}\delta/r \tag{6}$$

where r is the "day radius" (cf. Fig. 6) of the Sun. Thus

$$\operatorname{Sin}\omega = \frac{R\operatorname{Sin}\delta}{r}\cdot\frac{s_0}{g} \tag{7}$$

For geographical latitudes near 24° the quantities ω can be considered to be small angles for which the approximation

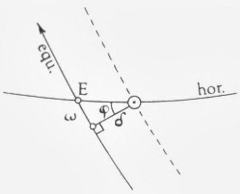

Fig. 5.

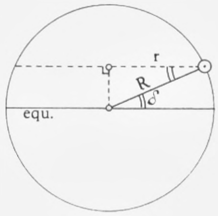

Fig. 6.

$$\operatorname{Sin}\omega \approx 2\omega^\circ \tag{8}$$

is permissible.[1]) Thus from (7)

$$2\omega^\circ \approx \frac{R\operatorname{Sin}\delta}{r}\cdot\frac{s_0}{g}$$

with $R = 120$ and $g = 12$, $d = 2r$:

$$2\,\omega^\circ = \frac{20\operatorname{Sin}\delta}{d}\cdot s_0 \quad \text{hence} \quad 2\omega^{\text{vin}} = \frac{200\operatorname{Sin}\delta}{d}\cdot s_0. \tag{9}$$

We do not know the exact values used for the declination δ and for d but the proper order of magnitude is found when using the parameters given in IV,23–25 (cf. Table 7). The values of the coefficients γ_i in (3) agree well enough with the result of our computation to leave little doubt as to the correctness of our reconstruction.

III,12. The region between the Himālayas and the ocean is Bhāratavarṣa, the Indian sub-continent, corresponding to latitudes φ roughly between 10° and 30°. For this

[1]) This approximation implies two more assumptions:

(a) $\pi \approx 3$ (b) $R = 120$.

For small α, counted in radians, we have $\sin\alpha \approx \alpha$, thus $\operatorname{Sin}\alpha \approx R\alpha$ and consequently $(\alpha R)^\circ \approx \frac{180}{\pi}\operatorname{Sin}\alpha \approx 60\operatorname{Sin}\alpha$ because of (a) and finally $2\alpha^\circ \approx \operatorname{Sin}\alpha$ because of (b).

area φ is sufficiently near to $\epsilon \approx 24°$ to justify the approximation (8) in III,10–11. The reference for the further explanations is to XIV,1–4.

III,13. The true distances of Avantī (= Ujjayinī) and Vārāṇasī (= Benares) from Yavanapura, assuming that the last is Alexandria in Egypt, are 45;50° and 53;7°. The distances given in the text are $0;7,20^d = 44°$ and $0;9^d = 54°$ respectively.

A result of this type, if not purely accidental, could only have been obtained from the observation of lunar eclipses. The procedure described in the next verse is not applicable for larger distances.

TABLE 7.

i	$\mathrm{Sin}\,\delta_i$	d_i	$\frac{200\,\mathrm{Sin}\,\delta_i}{d_i}$	$\frac{2(\omega_i - \omega_{i-1})}{s_0}$	γ_i
0	0	$4,0 = 2R$	0	0	
1	24;24	3,55	20;46	$20;46^{vin}$	20^{vin}
2	42;15	3,44;40	37;37	16;51	16;30
3	48;48	3,39;15	44;31	6;54	6;45

III,14. If d is the shortest distance between two localities, measured in yojanas, while their geographical latitudes differ by Δb degrees, the longitudinal difference, in degrees, is found from

$$\Delta l = \sqrt{(9\,d/80)^2 - \Delta b^2}$$

and hence the difference of local time $\Delta l/6$ nāḍikās.

The factor 9/80 is based on assuming 3200 yojanas for the equatorial circumference of the earth, hence

$$\frac{360°}{3200} = \frac{9°}{80}.$$

The same parameter is found in IX,10 and in XIII,15–16.

III,15. A fragmentary passage which in the present form makes no sense, e.g., because one cannot add longitudinal differences and ascensional differences.

III,16. Cf. II,7 and p. 22.

III,17. The daily velocity of the Sun as function of its longitude is assumed to be $1^{o/d}$ plus a small correction, constant in each sign (cf. the graph Fig. 7). The apogee lies correctly in Gemini if we assume that the enumeration begins with Aries. The arithmetical mean of the twelve given values is $0;59^{o/d}$, the same as the mean value between the minimum $0;57^{o/d}$ and the maximum $1;1^{o/d}$.

Obviously there is no astronomical reason for the asymmetry of the given values nor can such data be the result of direct observations. On the other hand, the text does not allow for plausible emendations. Hence it is probably a distorted tradition on which the extant text is based. It is, of course, not difficult to reconstruct a symmetric distribution, without changing either the extrema or the mean value. The dotted graph in Fig. 7, changing only two numbers by 0;1 respectively leads, e.g., to a typical "System B" type[1]) of distribution.

III,18–19. A karaṇa is half of a tithi or the period in which the elongation of the Moon from the Sun increases by $6° = 360'$.

Four of the 60 karaṇas thus contained in one synodic month are called "fixed" because the same name is always associated with them: the first karaṇa after conjunction, $n = 1$, is Kiṃstughna, and at the end $n = 58$ is Śakuni, $n = 59$ is Catuṣpada, $n = 60$ is Nāga. The remaining 56 karaṇas are called "movable" and are divided

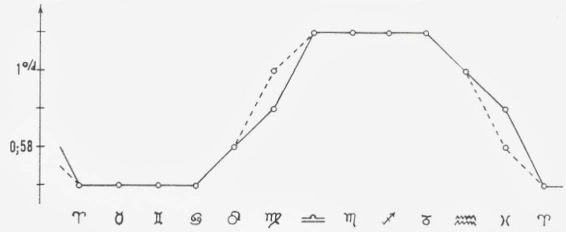

Fig. 7.

into 8 series of 7 each, named in order Bava, Bālava, Kaulava, Taitila, Gara, Vaṇij, and Viṣṭi (see Bṛhatsaṃhitā 99,4–5).

Consequently, in the first half of the month, in the śuklapakṣa, the second karaṇa has the name Bava, i.e., the number $k = 1$ among the movable karaṇas and in general

$$k \equiv n - 1 \mod 7 \tag{1}$$

where n is the consecutively counted number of the karaṇa (i.e., $n = 1, 2, \ldots, 60$) and k the order number of the names of the movable karaṇas (i.e., beginning with Bava, $k = 1$ and ending with Viṣṭi, $k = 7$).

In the second half of the month, i.e., in the kṛṣṇapakṣa, the first karaṇa has the number $n = 31$ but $k = 2$. Hence, if we count the karaṇas of the kṛṣṇapakṣa from $n' = 1$ to 30 we have now

$$k \equiv n' + 1 \mod 7. \tag{2}$$

In order to find n in the śuklapakṣa one takes the elongation $\lambda_m - \lambda_s = \Delta\lambda°$ where λ_m and λ_s are the longitudes of Moon and Sun respectively, reckoned in minutes, $\Delta\lambda°$ the corresponding elongation in degrees. We furthermore assume that these elongations refer to the upper end of a karaṇa such that, e.g., $\Delta\lambda° = 6°$ for the first karaṇa ($n = 1$). With these assumptions one can express (1) in the form

$$k = \frac{\Delta\lambda°}{6} - 1 = \frac{(\lambda_m - \lambda_s) - 360'}{360}. \tag{1a}$$

[1]) Cf., e.g., Neugebauer, Exact Sci.(²) p. 159.

For n' in the kṛṣṇapakṣa we have only to replace $\Delta\lambda°$ by $\Delta\lambda'° = \Delta\lambda° + 180°$ and obtain from (2)

$$k = \frac{\Delta\lambda'°}{6} + 1 = \frac{(\lambda_m + 180° - \lambda_s) + 360'}{360}. \tag{2a}$$

III,20–22. The vaidhṛta occurs when the Sun and the Moon are equidistant from and on opposite sides of an equinox; then

$$\lambda_s + \lambda_m = 360°. \tag{1}$$

The vyatipāta occurs when the Sun and Moon are equidistant from and on opposite sides of a solstice; then

$$\lambda_s + \lambda_m = 180°. \tag{2}$$

This simple scheme is modified by the assumption of a trepidation of the solstices and equinoxes over an arc of 46;40°. The summer solstice is assumed by Varāhamihira to be at Cancer 0° in his own time, in the middle of the arc of trepidation.[1]) But the summer solstice was once in the middle of Āśleṣā (i.e. Cancer 23;20°), and again it will be at Gemini 6;40°. At this latter extreme the vyatipāta produces, instead of (2), the sum

$$\lambda_s + \lambda_m = 180° - 46;40° = 133;20°.$$

This implies that the base vyatipāta occurred when the summer solstice was at Cancer 23;20°.

The choice of an arc of 23;20° is here associated with the obliquity of the ecliptic, though elsewhere that is stated to be 23;40° (IV,16–18) or 24° (IV,24). In fact the choice of the arc must be motivated by the statement in the Jyotiṣavedāṅga (Ṛk 6 = Yajus 7) that the northern ayana of the Sun begins at Dhaniṣṭhā, i.e., at Capricorn 23;20°.

Varāhamihira states in the Bṛhatsaṃhitā (2,1):

> āśleṣārdhād dakṣiṇam uttaram ayanaṃ raver dhaniṣṭhādyam/
> nūnaṃ kadācid āsīd yenoktaṃ pūrvaśāstreṣu//

"Once, according to what is said in ancient treatises, the southern ayana of the Sun was from the middle of Āśleṣā, and the northern began with Dhaniṣṭhā."

The Jyotiṣavedāṅga placed the beginning of the Sun's northern ayana in Dhaniṣṭhā because its list of nakṣatras began with Kṛttikā (as do those in the Atharvaveda and in the Samhitās) rather than with Aśvinī. The true difference between the two starting points is 26;40° not 23;20°.

[1]) As is usual in this and in other texts of Indian astronomy, all longitudes are sidereal, save in cases in which the declination of the Sun plays a role. For the sidereal coordinates the beginning of Aries is identical with the beginning of the nakṣatra Aśvinī.

III,23–24. The ṣaḍaśītimukhas divide the ecliptic into four equal arcs of 86° each and one remaining arc of 16°. The four equal arcs are:

Libra 0° to Sagittarius 26°, Sagittarius 26° to Pisces 22°,
Pisces 22° to Gemini 18°, Gemini 18° to Virgo 14°.

The remaining arc from Virgo 14° to Libra 0° completes the circle. The modern Sūryasiddhānta (XIV,6) states that one must sacrifice to the Pitṛs on the 16 days that do not fall in a ṣaḍaśītimukha; the reason for this practice is not mentioned.

III,25. For the seasons or ṛtus see Part I p. 187. The expression is reminiscent of the passage in the Parāśaratantra cited by Utpala (on Bṛhatsaṃhitā 2,1): "While the Sun travels from the beginning of Śraviṣṭhā (= Dhaniṣṭhā) to the end of Pauṣṇa (= Revatī) it is Śiśira; from the end of Pauṣṇa to the end of Rohiṇī, Vasanta; from the beginning of Saumya (= Mṛgaśiras) to the middle of Sārpa (= Āśleṣā), Grīṣma; from the middle of Sārpa to the end of Hasta, Prāvṛṭ (= Vārṣa); from the beginning of Citrā to the middle of Indra (= Jyeṣṭhā), Śarat; and from the middle of Jyeṣṭhā to the end of Vaiṣṇava (= Śravaṇa), Hemanta."

Parāśara adheres to the old scheme of the Jyotiṣavedāṅga. Varāhamihira's verse is intended to bring the calendar in line with his solstice position.

III,26. The problem is to find the time in nāḍīs that it takes the disc of the Sun to traverse a point on the ecliptic. If v_s is the daily progress of the Sun in minutes, then $v_s/60$ is the motion during one nāḍī. Consequently it takes $60\, d_s/v_s$ nāḍīs for the Sun to move the d_s minutes of its apparent diameter. The mean diameter of the Sun is stated in VIII,13 to be 0;30°.

This solution is specifically applied to the saṅkrāntis or entries of the Sun into a zodiacal sign, which are auspicious. By computation of the Sun's longitude the moment of the entry of its center into a sign is determined; the period of auspiciousness is assumed to extend over the length of time determined in this verse, half taken before, half after the computed moment.

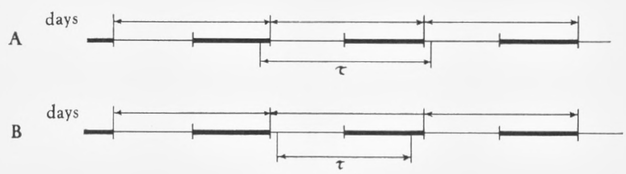

Fig. 8A–B.

III,27. Three possibilities exist: that a true tithi (τ) be longer than the sāvana (sunrise) day, that it be equal to it, or that it be shorter than it. For the first case Fig. 8A illustrates the situation when the tithi participates in three days while Fig. 8B shows for a short tithi the case in which one day participates in three tithis.

III,28–29. These verses concern the retrograde motion of the lunar nodes. Since the ahargaṇa a is to be multiplied by 8/151 we have in first approximation a daily motion of the nodes by

$$8/151 = 8/2,31 \approx 0;3,10,43,42,31,\ldots{}^{\circ/d}.$$

At this rate one revolution requires exactly

$$2,31 \cdot 6,0/8 = 1,53,15^{d}.$$

Consequently the number of revolutions of the nodes during a days is $a/1,53,15$. For each revolution a correction of $+1'$ of retrograde motion is required. Thus the daily motion is given by[1])

$$\frac{8}{2,31} + \frac{1}{1,53,15,0} \approx 0;3,10,43,42,31 + 0;0,0,0,31,47 \approx 0;3,10,44,14,18,\ldots{}^{\circ/d}.$$

In III,29 the ascending node at epoch is placed at Scorpio 25;59° ($\lambda = 235;59°$). Modern tables[2]) give for 505 March 22 for the ascending node about Scorpio 25;54°. Varāhamihira's Sūryasiddhānta would place it at Scorpio 26;6,57°; see below IX,5.

III,30–31. The variation of the lunar latitude is treated as a simple linear zigzag function with 4;40° as maximum, a value which seems to be attested nowhere else. The value in IX,6 is the more common one 4;30°.

III,32–35. These verses are evidently based on some obscure speculation in the Romakasiddhānta about the duration of creation. III,33 seems to belong naturally with XV,17–27.

The separation of tithi and nakṣatra presumably means that at the first tithi of the month the Moon is not in the first nakṣatra, Aśvinī; this separation is supposed to be an auspicious muhūrta for the pratipatti, i.e. the beginning of any action (or the beginning of creation?). However, if on a bhadrā tithi (the 2nd, 7th, or 12th in any pakṣa[3])) the Moon is in Śravaṇa (Sagittarius 10° to 23;20°), the muhūrta is inauspicious. The inauspiciousness arises from the fact that the creation ceases at such a yuga, i.e. when the conjunction of the Sun and Moon (the first tithi) occurs in Uttarāṣāḍhā, i.e. at the winter solstice. This is reminiscent of Hellenistic speculations regarding a "world-year."[4])

The 68 550 years in III,34 is derived from the Romakasiddhānta; it is equal to $24 \cdot 19 \cdot 150 + 150 = 19,2,30$, where $19 \cdot 150 = 2850$ years is the Romaka's yuga (cf. I,15). The significance of this computation is obscure.

[1]) Cf. also IX,5.
[2]) Ernest W. Brown, Tables of the Motion of the Moon (New Haven 1919) Section II.
[3]) The 15 tithis of every half-month are divided into three equal series whose members are consecutively called mandā, bhadrā, vijayā, riktā, and pūrṇā; see Bṛhatsaṃhitā 99,2.
[4]) See e.g., B. L. van der Waerden, Hermes 80 (1952) p. 129–155.

The meaning of III,35 also defies comprehension. Dikshit [1890a] has indeed demonstrated that, by the elements of Varāhamihira's Sūryasiddhānta, the Caitra whose pratipad is used as epoch in this karaṇa is pūrṇimānta; but there is no reason to compute the longitudes of the Sun and Moon for the pūrṇimā of that month. Moreover, at Caitrapūrṇimā the Moon must be close to Libra 0° so that the Moon on the ninth tithi is far from Punarvasu (Gemini 20° to Cancer 3;20°). The reference to Punarvasu rather suggests an ecpyrosis at the summer solstice as we had a cataclysm at the winter solstice (III,32), but the text as it stands does not allow us to arrive at this interpretation.

III,36–38. The need to select the proper times (muhūrta) for the twice-born (Brāhmaṇa, Kṣatriya, or Vaiśya) to perform the various śrauta (Vedic) and smārta (customary) rituals is one of the principal motivations for the study of astronomy in India. Therefore those who despise astronomy as being inaccurate as well as those who incorrectly practice it are suitably punished, while experts in the field are rewarded in the traditional manner. Compare IX,8 in the Paitāmahasiddhānta of the Viṣṇudharmottarapurāṇa.

Chapter IV

IV,1–5. Let c be the circumference of a circle of radius R, d its diameter. It is assumed that

$$d = \sqrt{c^2/10} \tag{1}$$

hence

$$\pi = \sqrt{10} \approx 3.162 \quad \text{instead of} \approx 3.141. \tag{1a}$$

Furthermore

$$\left.\begin{aligned} \text{Sin } 30° &= \sqrt{R^2/4} = R/2 \\ \text{Sin } 60° &= \sqrt{R^2 - R^2/4} = R\sqrt{3}/2. \end{aligned}\right\} \tag{2}$$

Finally a formula is given to obtain $\text{Sin}\,\alpha$ from $\text{Sin}\,2\alpha$, i.e., to go from Sin 30° to Sin 15°, Sin 7;30°, Sin 3;45°:

$$\text{Sin}^2\alpha = \left(\frac{R - \text{Sin}(90-2\alpha)}{2}\right)^2 + \left(\frac{\text{Sin}\,2\alpha}{2}\right)^2. \tag{3}$$

This formula can easily be obtained from the relation between chord- and sine-function (cf. Fig. 9):

$$\text{Crd}^2\theta = \text{Vers}^2\theta + \text{Sin}^2\theta$$

and

$$\text{Crd}\,\theta = 2\,\text{Sin}\frac{\theta}{2}$$

give indeed

$$4\,\text{Sin}^2\frac{\theta}{2} = (R - \text{Cos}\,\theta)^2 + \text{Sin}^2\theta.$$

The relation (3) is the equivalent of

$$4\sin^2\alpha = (1 - 2\cos 2\alpha + \cos^2 2\alpha) + \sin^2 2\alpha = 2 - 2\cos 2\alpha$$

hence

$$2\sin^2\alpha = 1 - \cos 2\alpha. \tag{4}$$

Also this form is found in our text (IV,5):

$$\mathrm{Sin}^2\alpha = 60(R - \mathrm{Sin}(90 - 2\alpha)) \tag{5}$$

which follows from (4) if $R = 120$.

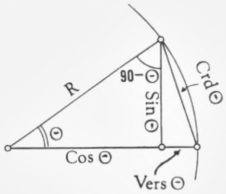

Fig. 9.

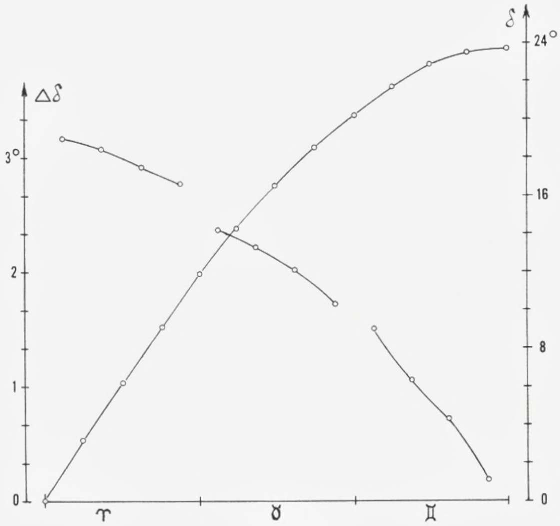

Fig. 10.

IV,6–15. Table 8 shows the numerical equivalents of the numbers described in these verses. The units are called "minutes" and "seconds" but in fact the "minutes" are the same units in which R counts as 120. The terminology is derived from the earlier Sine-table in the Paitāmahasiddhānta of the Viṣṇudharmottarapurāṇa (III,12) in which the radius as well as the circumference are reckoned in minutes ($R = 3438 = 57{,}18$, $c = 21\,600 = 6{,}0{,}0$).

IV,16–18. These verses concern the solar declinations (δ) for which the text gives the differences ($\Delta\delta$) in steps of $7;30°$ of longitude (cf. Table 9). The trend of these differences is rather irregular (cf. Fig. 10) but the effect on the declinations themselves is not very visible. The total for 90° is

$$\varepsilon = 23;40°. \tag{1}$$

and that this was intentional is shown by the statement in IV,16 that

$$\mathrm{Sin}\,\varepsilon = 48;9 \tag{2}$$

which is indeed correct for $\varepsilon = 23;40°$.

From IV,23–25 one derives the round value $\varepsilon = 24°$.

TABLE 8.

α	$\sin\alpha$	$\mathrm{Sin}_{120}\,\alpha$	Text $\mathrm{Sin}_{120}\,\alpha$	Δ	Text Δ
3;45°	0; 3,55,27	7;50,54	7;51	7;50,54	7;51
7;30	7,49,54	15;39,48	15;40	7;48,54	7;49
11;15	11,42,20	23;24,40	23;25	7;44,52	7;45
15	15,31,45	31; 3,30	31; 4(+)	7;38,50	7;39
18;45	19,17,11	38;34,22	38;34	7;30,52	7;30(−)
22;30	22,57,40	45;55,20	45;56(+)	7;20,58	7,22(+)
26;15	26,32,14	53; 4,28	53; 5(+)	7; 9, 8	7; 9
30	30	1, 0	1, 0	6;55,32	6;55(−)
33;45	33,20, 3	1, 6;40, 6	1, 6;40	6;40, 6	6;40
37;30	36,31,32	1,13; 3, 4	1,13; 3	6;22,58	6;23
41;15	39,33,39	1,19; 7,18	1,19; 7	6; 4,14	6; 4
45	42,25,35	1,24;51,10	1,24;51	5;43,52	5;44
48;45	45, 6,37	1,30;13,14	1,30;13	5;22, 4	5;22
52;30	47,36, 4	1,35;12, 8	1,35;12	4;58,54	4;59
56;15	49,53,17	1,39;46,34	1,39;46(−)	4;34,26	4;34
60	51,57,41	1,43;55,22	1,43;55	4; 8,48	4;11(+)
63;45	53,48,45	1,47;37,30	1,47;37(−)	3;42, 8	3;42
67;30	55,25,58	1,50;51,56	1,50;52	3;14,26	3;15(+)
71;15	56,48,57	1,53;37,54	1,53;37(−)	2;45,58	2;45(−)
75	57,57,20	1,55;54,40	1,55;55	2;16,46	2;18(+)
78;45	58,50,50	1,57;41,40	1,57;42	1;47, 0	1;47
82;30	59,29,12	1,58;58,24	1,58;59(+)	1;16,44	1;17
86;15	59,52,18	1,59;44,36	1,59;44(−)	0;46,12	0;45(−)
90	1; 0, 0, 0	2, 0; 0, 0	2, 0; 0	0;15,24	0;16(+)

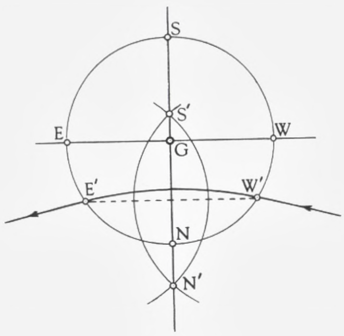

Fig. 11.

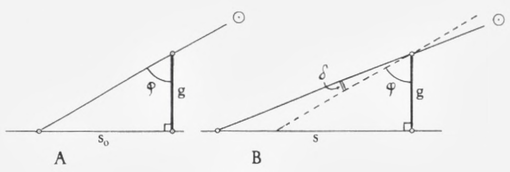

Fig. 12.

TABLE 9.

λ	$\Delta\lambda$		δ
7;30°	190′ =	3;10°	3;10°
15	183	3; 3	6;13
22;30	175	2;55	9; 8
30	166	2;46	11;54
37;30	142′ =	2;22°	14;16
45	133	2;13	16;29
52;30	121	2; 1	18;30
60	103	1;43	20;13
67;30	90 =	1;30°	21;43
75	63	1; 3	22;46
82;30	43	0;43	23;29
90	11	0;11	23;40

IV,19. Construction of the cardinal directions in a horizontal plane (cf. Fig. 11) If G is the foot of a gnomon, E′, W′ the intersections of a shadow-curve with a circle of center G (why of radius $4g$?); perpendicular to E′W′ (based on a "barley-corn figure" N′S′) is the N–S direction.

IV,20–23. The following formulae are direct consequences of Fig. 12A and B, assuming $R = 120$ and $g = 12$:

$$\mathrm{Sin}\,\varphi = 120\, s_0/\sqrt{s_0^2 + 144} \tag{1}$$

$$s = 12\ \mathrm{Sin}(\varphi \pm \delta)/\sqrt{R^2 - \mathrm{Sin}^2(\varphi \pm \delta)} \tag{2}$$

$$\mathrm{Sin}\,\bar{\varphi} = \sqrt{R^2 - \mathrm{Sin}^2 \varphi}\,. \tag{3}$$

The second part of IV,21 implies that, when one replaces the equinoctial noon shadow s_0 in (1) by s, the noon shadow at any day, one obtains the Sine of the Sun's coaltitude $\bar{h}$:

$$\mathrm{Sin}\,\bar{h} = Rs/\sqrt{s^2 + g^2} \tag{1a}$$

and again φ from $\bar{h}$ by adding (algebraically) the declination:

$$\varphi = \bar{h} + \delta. \tag{1b}$$

IV,23–25. It follows from Fig. 6 (p. 30) that the radius r of the Sun's daycircle at a declination δ is given by

$$r = \sqrt{R^2 - \mathrm{Sin}^2 \delta} \tag{1}$$

measured in units in which $R = 120$. In the text we have the following numerical relations

$$\left.\begin{array}{rccc} \lambda_s: & 30^\circ & 60^\circ & 90^\circ \\ \text{Sin}\,\delta: & 24;24 & 42;15 & 48;48 \\ 2r: & 3,55 & 3,44;40 & 3,39;15. \end{array}\right\} \quad (2)$$

It follows from (1) that one should have $\text{Sin}^2\delta + r^2 = R^2 = 4,0,0$. In fact one obtains from (2) for $\text{Sin}^2\delta + r^2$ the values

$$3,54,51;\ldots \quad 4,0,3;\ldots \quad 3,59,58;\ldots$$

respectively. Since the last number agrees best we may use correspondingly

$$48;48 = \text{Sin}\,\varepsilon \tag{3}$$

for the determination of the obliquity ε of the ecliptic and find by linear interpolation in Table 8 (p. 38) of IV,6–15[1])

$$\varepsilon = 24;0,12,\ldots \approx 24^\circ \tag{4}$$

a much used round value, but different from the value used in IV,16–18.

IV,26. The following rule is given for the determination of the ascensional differences ω (cf. Fig. 5 p. 30):

$$2\omega^{\text{vin}} = 60\,\frac{\text{Sin}\,\varphi\cdot R\cdot\text{Sin}\,\delta}{\text{Sin}\,\bar{\varphi}\cdot d\cdot 3}. \tag{1}$$

Its correctness follows from (6) in III,10–12 (with $R = 120$, $g = 12$, $d = 2r$):

$$2\omega^{\text{vin}} = \frac{200\ \text{Sin}\,\delta}{d}\,s_0 = 10\cdot 12\ \tan\varphi\cdot\frac{60\ \text{Sin}\,\delta}{3d} = R\tan\varphi\cdot\frac{60\,\text{Sin}\,\delta}{3d}$$

q.e.d. Cf. also IV,34.

If we reckon ω in degrees instead of in vināḍikās we obtain from (1) simply

$$\omega = \frac{R}{r}\text{Sin}\,\delta\tan\varphi. \tag{2}$$

IV,27–28. Computing first the "earth-Sin" e from

$$e = d\ \text{Sin}\,\omega/240 \tag{1}$$

one finds φ from

$$\text{Sin}\varphi = eR/\sqrt{e^2+\text{Sin}^2\delta}. \tag{2}$$

The proof follows readily from Fig. 13:

$$e = \text{Sin}\,\delta\tan\varphi$$

and (cf. III,10–12 (7) and (5))

[1]) Accurate would be arc Sin 48;48 = 23;59,45°.

$$\mathrm{Sin}\,\omega = \frac{R}{r}\,\mathrm{Sin}\,\delta\,\tan\varphi$$

which gives (1) if $R = 120$. Fig. 13 shows furthermore that

$$\sin\varphi = e/\mathrm{AO} \qquad \mathrm{AO}^2 = e^2 + \mathrm{Sin}^2\delta$$

which proves (2).

IV,29–30. Let us assume that λ is a longitude within the first quadrant of the ecliptic; for the remaining quadrants obvious symmetries hold. Then the right ascensions $\alpha(\lambda)$ are determined by

$$\mathrm{Sin}\,\alpha = \frac{2R}{d}\sqrt{\mathrm{Sin}^2\lambda - \mathrm{Sin}^2\delta}. \tag{1}$$

If, in particular, α obtained from (1) is measured in degrees, then 10α is the right ascension measured in vināḍīs.

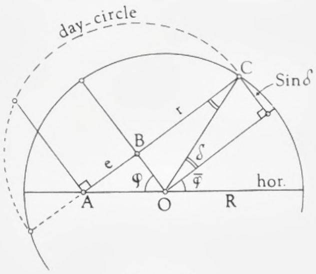

Fig. 13.

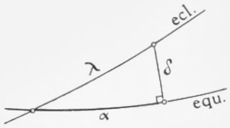

Fig. 14.

For the endpoints of the first three signs the text gives data which agree very well with the values found in the Almagest:

♈:	278^{vin} =	27;48°	Alm. II,8:	27;50°
♉:	299	29;54		29;54
♊:	323	32;18		32;16.

Proof of (1): It follows from Fig. 14 that[1])

$$\sin\varepsilon = \sin\delta/\sin\lambda \qquad \tan\varepsilon = \tan\delta/\sin\alpha \tag{2}$$

thus

$$\frac{\sin\alpha}{\sin\lambda} = \frac{\tan\delta}{\tan\varepsilon}\cdot\frac{\sin\varepsilon}{\sin\delta} = \frac{\cos\varepsilon}{\cos\delta} = \frac{r_0}{r} \tag{3}$$

where r_0 is the day-radius for $\lambda = 90°$ i.e. for $\delta = \varepsilon$ (cf. Fig. 13); thus

$$r_0 = R\cos\varepsilon. \tag{4}$$

[1]) It is not necessary to assume here the use of spherical trigonometry (Fig. 14) since the same relation can be readily obtained from an "analemma" (Fig. 15).

Hence from (3) and (2)

$$\operatorname{Sin}\alpha = \frac{r_0}{r}\operatorname{Sin}\lambda = \frac{R}{r}\operatorname{Sin}\lambda\cos\varepsilon = \frac{R}{r}\operatorname{Sin}\lambda\sqrt{1-\sin^2\varepsilon}$$

$$= \frac{R}{r}\operatorname{Sin}\lambda\sqrt{1-(\operatorname{Sin}^2\delta/\operatorname{Sin}^2\lambda)}$$

q.e.d.

IV,31. The difference between the right ascension $\alpha(\lambda)$ of a point of the ecliptic of longitude λ and the corresponding oblique ascension $\varrho(\lambda)$ is the quantity $\omega(\lambda)$ (cf. Fig. 5 p. 30):

$$\omega(\lambda) = \alpha(\lambda) - \varrho(\lambda) \tag{1}$$

which, in the terminology of the text, is half of the "ascensional difference" 2ω.

Since there is always one semicircle of the ecliptic above the horizon the rising time of λ equals the setting time of $\lambda + 180$.

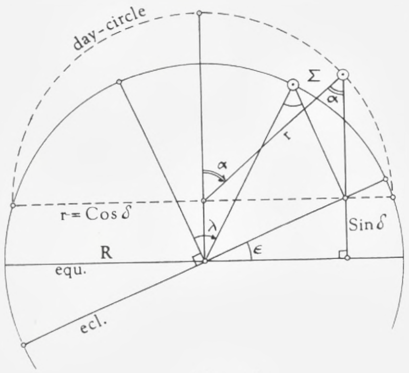

Fig. 15.

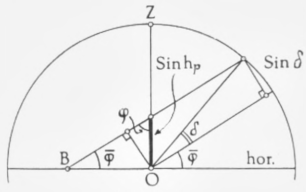

Fig. 16.

IV,32–33. When the Sun is on the northern semicircle (gola) of the ecliptic the declination δ is positive and one can compute the altitude h_p of the Sun in the prime vertical (cf. the projection on the meridian in Fig. 16) from

$$\operatorname{Sin} h_p = R\operatorname{Sin}\delta/\operatorname{Sin}\varphi. \tag{1}$$

Having found the altitude h_p of the Sun, the time (after sunrise or before sunset) when the Sun crosses the prime vertical can be computed, but the text does not tell us how this should be done.

IV,34. The increase of the length of daylight over 12 hours = 30 nāḍikās is given by

$$2\omega^n = \frac{2R\cdot\operatorname{Sin}\delta}{6\,d}\cdot\frac{\operatorname{Sin}\varphi}{\operatorname{Sin}\bar{\varphi}}. \tag{1}$$

This, in fact, is only a repetition of IV,26. It is here out of place since IV,35, 36, 38 return to the problems connected with the crossing of the prime vertical by the Sun.

IV,35–36. As in IV,32–33 we assume $\delta > 0$. Then

$$\operatorname{Sin} h_{\mathrm{p}} = \frac{\operatorname{Sin}\lambda \cdot \operatorname{Sin}\varepsilon}{\operatorname{Sin}\varphi}. \tag{1}$$

This follows immediately from (1) in IV,32–33 since with (2) in IV,29–30

$$R \operatorname{Sin}\delta = \operatorname{Sin}\lambda \operatorname{Sin}\varepsilon. \tag{2}$$

IV,37. By marking on the circle, whose center is the foot of the gnomon, the point that the gnomon-shadow at dawn intersects, one obtains the ortive amplitude by observation; from this can be computed δ by IV,39 and hence the longitude λ of the Sun.

IV,38. The Sun is in the prime vertical when the shadow of a vertical gnomon falls in the east-west line.

IV,39–40. The ortive- or setting-amplitude η of the Sun at declination δ can be found from

$$\operatorname{Sin}\eta = R \operatorname{Sin}\delta / \operatorname{Sin}\bar{\varphi}. \tag{1}$$

This follows from Fig. 16 because BO $= \operatorname{Sin}\eta$ and $\sin\bar{\varphi} = \operatorname{Sin}\delta/\mathrm{BO}$. Conversely (1) can be used to find $\bar{\varphi}$ and φ from

$$\operatorname{Sin}\bar{\varphi} = R \operatorname{Sin}\delta / \operatorname{Sin}\eta. \tag{2}$$

IV,41–44. Determination of the length of the shadow of a vertical gnomon at a given time t^{n} (in nāḍikās) after sunrise. From the given time

$$t^{\circ} = 6t^{\mathrm{n}} \tag{1}$$

and the ascensional difference found from IV,26 or IV,34

$$\omega^{\circ} = \tfrac{1}{20}\, 2\omega^{\mathrm{vin}} \tag{2}$$

one finds a time interval

$$t' = t^{\circ} \mp \omega^{\circ} \quad \text{if } \delta \gtrless 0. \tag{3}$$

From IV,23–25 the diameter of the day-circle is known, hence one can compute the altitude h of the sun from

$$\operatorname{Sin} h = \frac{d \operatorname{Sin}\bar{\varphi}}{2R^2}(\operatorname{Sin} t' \pm \operatorname{Sin}\omega) \tag{4}$$

and thus the length of the shadow

$$s = \frac{g}{\operatorname{Sin} h}\sqrt{R^2 - \operatorname{Sin}^2 h} \tag{5}$$

where for $R = 120$

$$2R^2 = 28\,800 \qquad R^2 = 14\,400.$$

The proof follows from Fig. 17 which represents the plane of the meridian with half of the day-circle turned into it, the Sun being at Σ. Then

$$\Sigma' B = \Sigma' \mathrm{C} \pm \mathrm{CB} = r \sin t' \pm r \sin\omega = \frac{r}{R}(\mathrm{Sin}\, t' \pm \mathrm{Sin}\,\omega) \qquad \delta \gtrless 0. \tag{6}$$

Furthermore

$$\mathrm{Sin}\, h/\Sigma' \mathrm{B} = \mathrm{Sin}\bar{\varphi}/R \tag{7}$$

which gives (4) using (6). Formula (5) means simply (cf. Fig. 18):

$$\cot h = s/g = \sqrt{R^2 - \mathrm{Sin}^2 h}/\mathrm{Sin}\, h. \tag{8}$$

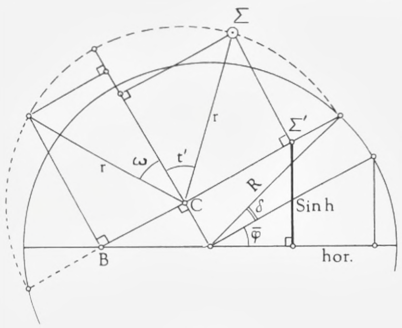

Fig. 17.

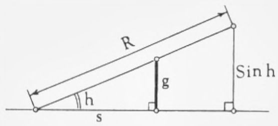

Fig. 18.

IV,45–47. This is the inverse problem to IV,41–44: find from the length s of the shadow the time t^{n} after sunrise. One is directed to compute to given s a "first Sine" S_1:

$$S_1 = \frac{gR^2}{\mathrm{Sin}\bar{\varphi}\sqrt{s^2+g^2}} \qquad gR^2 = 12 \cdot 120^2 = 172\,800 \tag{9}$$

and from it

$$S_2 = S_1 \mp \frac{\mathrm{Sin}\,\varphi}{\mathrm{Sin}\,\bar{\varphi}}\, \mathrm{Sin}\,\delta. \tag{10}$$

Then

$$t' = \mathrm{arc\ Sin}\left(S_2 \cdot \frac{2R}{d}\right) \tag{11}$$

and

$$\omega = \mathrm{arc\,Sin}\left(\frac{\mathrm{Sin}\,\varphi}{\mathrm{Sin}\,\bar{\varphi}}\, \mathrm{Sin}\,\delta \cdot \frac{2R}{d}\right) \tag{12}$$

from which the time after sunrise in nāḍikās

$$t^{\mathrm{n}} = \tfrac{1}{6}(t' + \omega). \tag{13}$$

Indeed, from (7) and Fig. 18 (or IV,52)

$$S_1 = \frac{R}{\operatorname{Sin}\bar{\varphi}} \cdot \frac{gR}{\sqrt{s^2+g^2}} = \frac{R}{\operatorname{Sin}\bar{\varphi}} \operatorname{Sin} h = \Sigma' \mathrm{B} \tag{14}$$

which explains (9). Consequently (cf. Fig. 17) for $\delta \gtrless 0$

$$S_2 = \Sigma' \mathrm{B} \mp \tan\varphi \operatorname{Sin}\delta = \Sigma' \mathrm{B} \mp e = \Sigma' \mathrm{C} = r \sin t' = \frac{r}{R} \operatorname{Sin} t'$$

therefore (11):

$$\operatorname{Sin} t' = S_2 \frac{2R}{d}.$$

Finally (Fig. 17)

$$r \sin\omega = \frac{r}{R} \operatorname{Sin}\omega = e = \tan\varphi \operatorname{Sin}\delta$$

therefore

$$\operatorname{Sin}\omega = \tan\varphi \operatorname{Sin}\delta \cdot \frac{2R}{d}$$

q.e.d.

IV,48–49. The relations developed in IV,41–47 for finding the time after sunrise (before noon) from the length of the shadow, and vice versa, are astronomically exact. What now follows is a crude approximation for the same problem, based on the simple assumption that the length s of the shadow at the time t after sunrise is given by

$$s = \frac{a}{t} + b \tag{1}$$

with arbitrary constants a and b. This relation satisfies at least the condition $s = \infty$ for $t = 0$ and a/t is the simplest function to give this result.

If C is the length of daylight, measured in the same units as t, then it follows from (1) that the noon shadow s_0 is given by

$$s_0 = \frac{2a}{C} + b.$$

Hence we have now instead of (1)

$$s = \frac{a}{t} + s_0 - \frac{2a}{C} \tag{2}$$

where a is still an arbitrary constant. For no special reason the text chooses

$$a = 6C, \tag{3}$$

hence IV,49:

$$s = \frac{6C}{t} + s_0 - 12 \tag{4}$$

and solving for t, IV,48:

$$t = \frac{6C}{s - s_0 + 12}. \tag{5}$$

IV,50–51. General remarks concerning the adaptation of the preceding rules to the Moon and the planets. In fact without value and out of place.

IV,52–54. It is supposed to be known: the solar longitude λ, the solar declination δ, the geographical latitude φ, the length s of the shadow ($g = 12$), and the E-W-line in the plane of the horizon. One wishes to find the distance s' of the endpoint A of the shadow from the E-W-line (cf. Fig. 19 B).

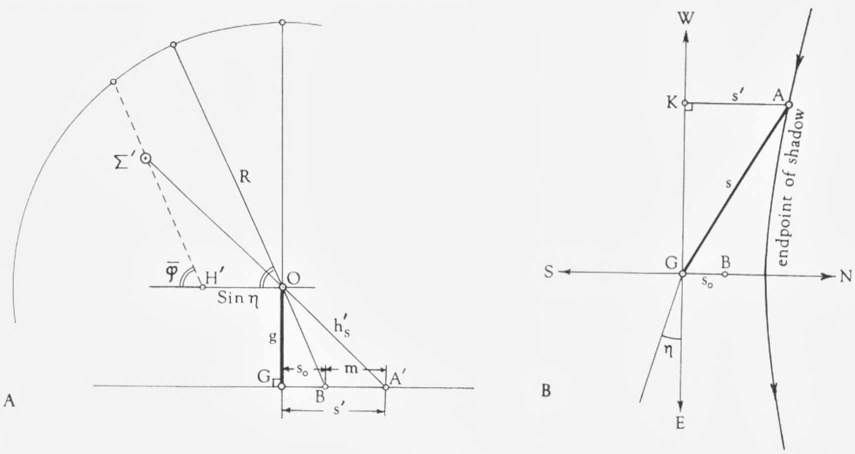

Fig. 19.

This problem is solved through the following steps. First one finds the altitude h of the Sun from

$$\operatorname{Sin} h = gR/\sqrt{s^2 + g^2} \tag{1}$$

(cf. Fig. 18 p. 44) and defines

$$\operatorname{Sin} \theta = \operatorname{Sin} h \frac{\operatorname{Sin} \varphi}{\operatorname{Sin} \bar{\varphi}}. \tag{2}$$

This angle θ is conventionally called the "Sun's amplitude" (sūryāgrā). Furthermore the ortive amplitude η of the Sun is given by

$$\operatorname{Sin} \eta = R \operatorname{Sin} \delta / \operatorname{Sin} \bar{\varphi} \tag{3a}$$

or

$$\mathrm{Sin}\,\eta = \mathrm{Sin}\,\varepsilon \cdot \mathrm{Sin}\,\lambda / \mathrm{Sin}\,\overline{\varphi} \tag{3b}$$

(cf. IV,39–40 (3) and IV,35–36 (2)). Then the "koṭi" s' is given by

$$s' = (\mathrm{Sin}\,\eta \mp \mathrm{Sin}\,\theta) h_s / R \quad \delta \gtrless 0 \tag{4}$$

where h_s is the "hypotenuse" of the shadow, i.e.,

$$h_s = \sqrt{s^2 + g^2}. \tag{5}$$

The distance GK on the east-west-line is called the "bāhu".

In order to prove (4) we consider in Fig. 19A the top O of the gnomon as the center of the celestial sphere and project the sun Σ onto Σ' in the meridian plane. Similarly $\mathrm{A'G} = s'$ is the projection of the shadow s, $h_s' = \mathrm{OA'}$ the projection of the hypotenuse OA. Let $s_0 = \mathrm{GB}$ be the equinoctial noon shadow, then

$$\mathrm{A'B} = s' - s_0 = m \qquad \mathrm{OH'} = \mathrm{Sin}\,\eta$$

and because

$$\frac{m}{h'} = \frac{\mathrm{Sin}\,\eta}{\mathrm{O}\Sigma'} \quad \frac{h'_s}{\mathrm{O}\Sigma'} = \frac{h_s}{\mathrm{O}\Sigma} = \frac{h_s}{R}$$

we have

$$m = \mathrm{Sin}\,\eta \cdot h_s / R. \tag{6}$$

Furthermore with (2), (1), and (5):

$$\mathrm{Sin}\,\theta \cdot h_s / R = \mathrm{Sin}\,h \cdot \tan\varphi \cdot h_s / R = \frac{gR}{h_s} \cdot \tan\varphi \cdot \frac{h_s}{R} = g \tan\varphi = s_0$$

hence

$$s' = m \mp s_0 \quad \delta \gtrless 0$$

as is correct according to Fig. 19A (drawn for $\delta < 0$).

IV,55–56. Here the inverse problem to IV,52–54 is solved: find the solar longitude λ from observed s and s', φ and ε being known, $g = 12$. With

$$h_s = \sqrt{s^2 + g^2}$$

one finds from (4)

$$\mathrm{Sin}\,\eta \mp \mathrm{Sin}\,\theta = Rs'/h_s. \tag{6}$$

From (1) and (2) one finds $\mathrm{Sin}\,\theta$, hence $\mathrm{Sin}\,\eta$ from (6) and $\mathrm{Sin}\,\lambda$ from (3b).

Chapter V

V,1–3. If $\varDelta\lambda$ is the elongation of the Moon from the Sun, $\varDelta\delta$ the difference of the respective declinations, β the lunar latitude, then we are told to compute

$$A = \sqrt{(\varDelta\lambda + \varDelta\delta)(\varDelta\lambda - \varDelta\delta)} = \sqrt{\varDelta\lambda^2 - \varDelta\delta^2} \tag{1}$$

and from it

$$B = \beta\varDelta\delta/A. \tag{2}$$

Finally one forms

$$C = \varDelta\lambda \mp B \quad \text{at first visibility} \tag{3a}$$

and

$$C = \varDelta\lambda \pm B \quad \text{at last visibility} \tag{3b}$$

where the upper sign is to be used when the latitude β has the same sign as $\varDelta\delta = \delta_{\mathrm{m}} - \delta_{\mathrm{s}}$, the lower sign in the opposite case.

For first visibility it is required that

$$\bar{\varrho}(C) \geq 2 \text{ nāḍikās} = 12^\circ \tag{4}$$

where $\bar{\varrho}(C)$ is the setting time of the arc C, i.e., the rising time of $C + 180^\circ$.

In order to explain these rules we consider the situation near sunset at first visibility (cf. Fig. 20), the Sun being at Σ, the Moon at M, in the case that

$$\left.\begin{aligned} \mathrm{U}\Sigma &= \varDelta\lambda = \lambda(\mathrm{M}) - \lambda(\Sigma) > 0 \\ \mathrm{UV} &= \varDelta\delta = \delta(\mathrm{U}) - \delta(\Sigma) > 0. \end{aligned}\right\} \tag{5}$$

Treating all triangles as plane we see that A in (1) represents the right ascensional difference

$$A = \varDelta\alpha = \alpha(\mathrm{V}) - \alpha(\Sigma) = \mathrm{V}\Sigma. \tag{6}$$

Furthermore

$$\varDelta\delta/A = \mathrm{UV}/\mathrm{U}\Sigma = \cot\theta = \mathrm{c}/\beta.$$

Therefore, from (2):

$$B = \beta\cot\theta = c = \mathrm{UT} \tag{7}$$

and from (3a):

$$C = \varDelta\lambda - c = \mathrm{U}\Sigma - \mathrm{UT} = \mathrm{T}\Sigma = \varDelta\lambda^* \tag{8}$$

(which, incidentally, is the "polar longitude" of the Moon). The corresponding right ascensional difference is the setting time, i.e., the delay of moonset over sunset

$$\bar{\varrho}(C) = \mathrm{R}\Sigma \tag{9}$$

which must exceed 12° in order to make the first crescent visible.

V,4–7. These verses concern the determination of the width σ (sūtra) of the illuminated sickle of the new crescent (cf. Fig. 21). We first give an astronomically plausible solution of the problem (cf. Fig. 22) and discuss only at the end how the erroneous formulation in the text may have originated.

Under all circumstances we may consider to be known the longitudes of Sun and Moon, hence also their elongation $\Delta\lambda$. Fig. 21 illustrates how the width of the sickle increases with $\Delta\lambda$ such that σ can be found from

$$\sigma = r_{\mathrm{m}}(1 - \cos\Delta\lambda) \tag{1}$$

where r_{m} is the apparent radius of the Moon. According to V,4 the apparent diameter of the Moon contains 15 "parts"; the origin of this division is unknown but it seems also to be used in XIV,38.

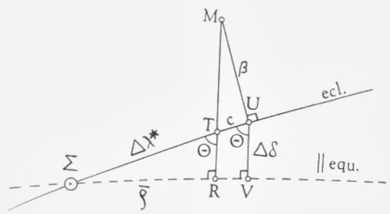

Fig. 20.

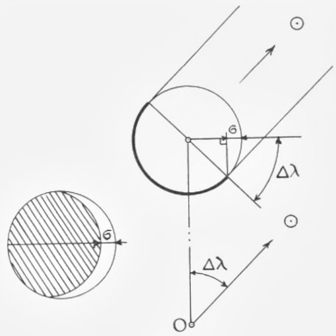

Fig. 21.

If one assumes that $\cos\Delta\lambda$ decreases linearly from 1 to 0 as $\Delta\lambda$ increases from $0°$ to $90°$ one can replace (1) by

$$\sigma = \frac{\Delta\lambda}{180} \cdot 15 = \frac{\Delta\lambda}{12} \tag{2}$$

measured in "parts". For $\Delta\lambda = 90°$, i.e. for quadrature, one obtains correctly $\sigma = 7;30 = r_{\mathrm{m}}$.

In (1) we have tacitly assumed that the Moon lies in the ecliptic. If, however, the Moon is at a latitude β, as shown in Fig. 22, then $\Delta\lambda$ has to be replaced in (1) and (2) by

$$h = \sqrt{\Delta\lambda^2 + \beta^2}. \tag{3}$$

This would solve our problem. The text, however, continues (cf. V,5) by forming the "koṭi" by combining β and the arc of declination $\Delta\delta$

$$k = \beta + \Delta\delta \tag{4}$$

(with proper signs) as shown in Fig. 22; finally the bāhu (or bhuja) is found from

$$b = \sqrt{h^2 - k^2}. \tag{5}$$

The reason for computing $b = \varDelta\alpha$ would be that $\varDelta\alpha \geqq 12°$ is the criterion for the visibility of the new crescent (cf. XVI,23 and XVII,58).

The formulation of the text (cf. Fig. 23) does not agree with this interpretation since it is said that the quantity σ (the sūtra) is to be laid off on the bhuja b (V,4) "which goes toward (the center of) the Moon" (V,7). It seems that h and b were interchanged, an error perhaps caused by the description of h as "the difference of Sun and Moon" (V,5) which would normally mean the longitudinal difference $\varDelta\lambda$ and not the actual distance of the luminaries that is needed in our interpretation.

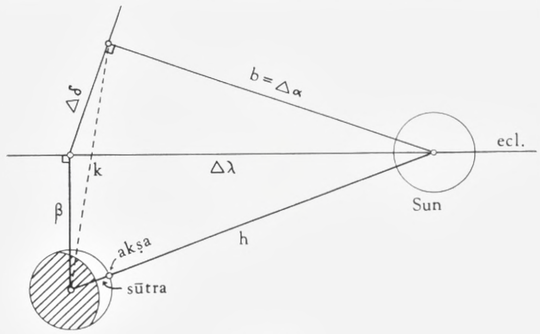

Fig. 22.

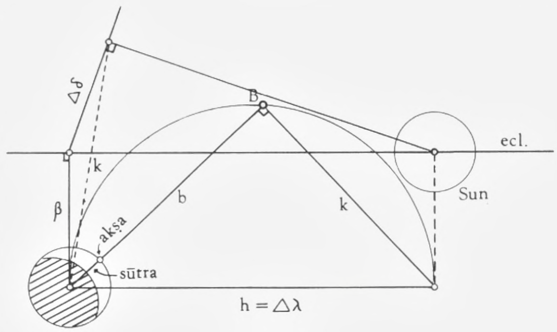

Fig. 23.

V,8–10. Let U be the point of the ecliptic for which $\lambda(\mathrm{U}) = \lambda(\mathrm{M})$, hence $\mathrm{U}\Sigma = \varDelta\lambda$ (cf. Fig. 20). The rising time of this arc (or the setting time) gives the delay of sunrise over moonrise, or of moonset over sunset, in V,9 or V,10 respectively.

In V,8 is computed a quantity d from

$$d = \beta \frac{s_0}{12} = \beta \tan\varphi \qquad (1)$$

where s_0 is the equinoctial noon shadow, $g = 12$ the length of the gnomon. It seems as if d should represent a correction of the rising times, needed for lunar latitudes $\beta \neq 0$. In fact such a correction should depend on λ as is correctly implied in V,1–3.

The influence of the lunar latitude will be expressed by (1) only if one identifies the ecliptic with the equator. In other words one may say that (1) is "in the mean" correct and gives at least the proper sign for the influence of the latitude. But it seems strange that so crude a rule is given after a correct solution of the problem had been found in V,1–3.

Chapter VI

VI,1. In order to find the moment of opposition of Sun and Moon it is assumed that the longitudes of the two luminaries are known (found by some standard method of computation) for the sunrise after opposition (assuming that the eclipse occurs during night time). Let λ_{mr} and λ_{sr} denote the longitudes of the Moon and the Sun respectively for the moment of sunrise and thus

$$\Delta\lambda = \lambda_{mr} - 180^\circ - \lambda_{sr} \tag{1}$$

the elongation of the Moon from the shadow center at sunrise. Assuming that $\Delta\lambda$ increases 12° per day, i.e. $0;12^\circ$ per nāḍī, the time Δt from opposition to sunrise will be given by

$$\Delta t = \frac{\Delta\lambda}{0;12} = 5\Delta\lambda \tag{2}$$

nāḍīs. The longitude λ_m of the Moon at opposition can then be computed for the moment Δt nāḍīs before sunrise.

VI,2. A lunar eclipse will take place if

$$|\,\lambda_m - \lambda'_c\,| < 13^\circ \tag{1}$$

where λ_m is the longitude of the Moon at opposition, and

$$\lambda'_c = \lambda_c + 0;36^\circ \tag{2}$$

λ_c being the longitude of the node near λ_m. The modification (2) of λ_c to λ'_c is probably introduced in order to formulate (1) in terms of integer degrees (again used in VII,5).

If

$$13^\circ < |\,\lambda_m - \lambda'_c\,| < 15^\circ \tag{3}$$

only a "darkening" takes place. It follows from VI,3 (cf. Fig. 24) that for the interval (3) the Moon does not at all come into contact with the earth's shadow. Hence it seems an inescapable conclusion that (3) refers to the penumbra though this concept seems to be otherwise unknown in ancient and mediaeval astronomy.

There is no trace of a recognition of the dependence of the eclipse limits on the lunar (and solar) anomaly.

VI,3. In this whole chapter the following dimensions are assumed for the apparent radii of the disk of the Moon and the earth's shadow:

$$r_{\mathrm{m}} = 0;17^\circ \qquad r_{\mathrm{u}} = 0;38^\circ \tag{1}$$

thus

$$r_{\mathrm{m}} + r_{\mathrm{u}} = 0;55^\circ \qquad r_{\mathrm{u}} - r_{\mathrm{m}} = 0;21^\circ. \tag{1a}$$

In the vicinity of the nodes all triangles are assumed to be plane and the lunar latitude β is reckoned as if perpendicular to the Moon's orbit (cf. Fig. 24) instead of to the ecliptic.

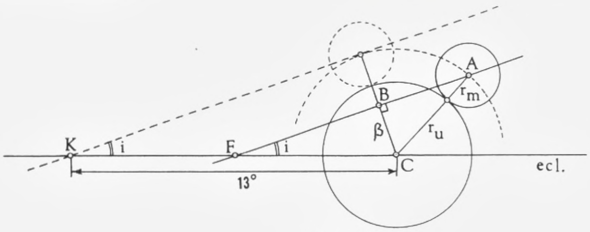

Fig. 24.

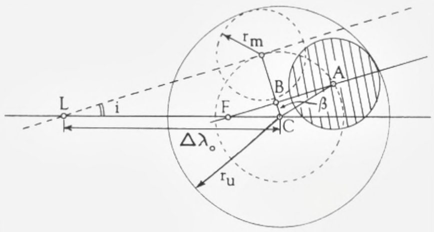

Fig. 25.

It follows from Fig. 24 that the duration Δt of a lunar eclipse is given by $2\mathrm{AB}/(v_{\mathrm{m}} - v_{\mathrm{s}})$ if $v_{\mathrm{m}} - v_{\mathrm{s}}$ represents the relative velocity of the Moon with respect to the Sun. Thus

$$\Delta t = \frac{2}{v_{\mathrm{m}} - v_{\mathrm{s}}} \sqrt{(r_{\mathrm{u}} + r_{\mathrm{m}})^2 - \beta^2}. \tag{2}$$

From the boundary 13;36° in VI,2 (1) and (2) it follows for the inclination i of the lunar orbit that

$$\sin i = 0;55/13;36 \approx 0;4,2,40,\ldots \tag{3}$$

Since $\sin 4^\circ \approx 0;4,11$ it is clear that

$$i = 4^\circ \tag{4}$$

was intended.[1]) This value would have to be lowered to 3;30° if the boundary of 15° in VI,2 (3) were still to produce an eclipse in the proper sense of the term; nor would it then be meaningful to give a second limit of 13° to which no characteristic phase would correspond.

VI,4. The arc $\Delta\lambda = 13^\circ - (\lambda_{\mathrm{m}} - \lambda_{\mathrm{c}})$ is multiplied by 5. Exactly as in VI,1 (2) the result represents a time interval Δt, reckoned in nāḍīs, and Fig. 24 shows that it is the time required for the elongation to increase by the arc FK. This Δt should be added to or subtracted from a "duration of the eclipse", depending on the position of the node before or behind the center of the shadow. What is meant by "duration" of the eclipse and what the result should represent is not clear to us.

[1]) Arcsin 0;4,2,40 = 3;52°. Also in VI,5 $i = 4^\circ$ is assumed.

VI,5. It follows from Fig. 25 that for $i = 4°$ (cf. VI,3 (4))

$$\sin i = 0;4,11 = (r_u - r_m)/\varDelta\lambda_0 = 0;21/\varDelta\lambda_0 \tag{1}$$

thus

$$\varDelta\lambda_0 = (r_u - r_m)/\sin i = 21/4;11 \approx 5;1 \approx 5°. \tag{2}$$

Furthermore

$$\beta = (\lambda_m - \lambda_c') \sin i = \varDelta\lambda \sin i$$

and

$$AB^2 = (r_u - r_m)^2 - \beta^2 = (\varDelta\lambda_0^2 - \varDelta\lambda^2) \sin^2 i.$$

If we express the duration 2AB° of totality in minutes of arc we have for it

$$\tau = 2{,}0 \text{ AB} = \text{Sin}\, i \sqrt{\varDelta\lambda_0^2 - \varDelta\lambda^2} \qquad R = 120.$$

From (1) and (2) we find

$$\text{Sin}\, i = 2 \cdot 21/5$$

hence

$$\tau = \frac{2 \cdot 21}{5} \sqrt{5^2 - \varDelta\lambda^2}. \tag{3}$$

The text gives this relation in the form

$$\tau = \tfrac{21}{5} \sqrt{4(5 - \varDelta\lambda)(10 - (5 - \varDelta\lambda))}$$

which is indeed the equivalent of (3).

VI,6. The duration of the partial phase of a lunar eclipse is the difference between the duration found in VI,3 and the duration of totality (VI,5).

VI,7. Cf. VI,12–15.

VI,8. The "deflection" γ referring to a given position of the Sun or the Moon (at a solar- or lunar-eclipse respectively) is the angle at the given point between the ecliptic and the "east-west-line" E′W′ (cf. Fig. 26) which is perpendicular to the great circle N′S′ through the eclipsed body and the north- and south-point on the horizon. This definition is not explicitly found in the present text but there seems to be no reason to doubt the identity of terminology with later texts, e.g. Raṅganātha's commentary on Sūryasiddhānta IV,24–25.

If we for the moment identify ecliptic and equator, denoting the culminating point as C (cf. Fig. 27), the lunar (or solar) longitude as λ, we can represent the rule given in the present verse for finding the deflection γ by

$$\gamma = 90°(\text{C} - \lambda)\varphi/1800'. \tag{1}$$

If we cancel the factor 90°/90 and call the distance between C and λ the hour angle t then we have instead of (1) the rule

$$\gamma = t\varphi/90. \tag{2}$$

The factor $t/90$ has the value 0 at the meridian and 1 at the horizon. Hence (2) implies that $\gamma = 0$ in the meridian and $\gamma = \varphi$ in the horizon as is indeed the case for the angle between east-west-line and equator at these extremal points. The rule (2) represents simply a linear interpolation between these limits.

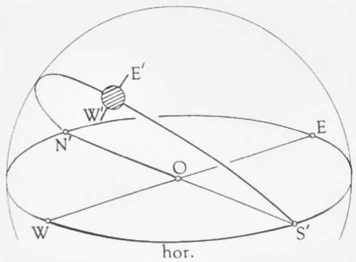

Fig. 26.

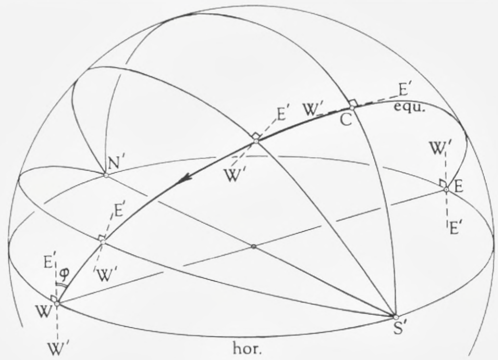

Fig. 27.

The values $\gamma = 0$ and $\gamma = \varphi$ are also obtained if we write

$$\gamma = \varphi \sin t \tag{3}$$

or even

$$\operatorname{Sin}\gamma = \operatorname{Sin}\varphi \operatorname{Sin} t/R \tag{4}$$

where a trigonometric interpolation replaces the linear relation (2). The rule (4) is found in XI,2. It is also given in the Āryabhaṭīya (Gola 45) and in the Khaṇḍakhādyaka (IV,7).

The preceding rule concerns only the computation of one component of the deflection, the "akṣavalana" which depends on φ of the locality and the t of the particular eclipse; it should further be modified by a second component, the "ayanavalana", in the amount of $\pm\delta(\lambda + 90°)$ as stated in XI,3. That at least one verse is missing after VI,8 is indicated by the fact that VI,9 is not found in our manuscripts, but is quoted with VI,10 by Utpala. In the same direction points also the fact that VI,9 introduces a new topic.

VI,9–10. The use of eclipse-colors as omens, which is found, e.g., in Bṛhatsaṃhitā 5,53–59, can be traced back to the Babylonian series Enūma Anu Enlil, tablets 15–18.[1]) The goal of the present verses is to establish criteria for predicting the colors. Varāhamihira's criteria are the altitude of the eclipsed body, its relation to ascendent or descendent, and the magnitude of the eclipse; normally Indians relate the colors to the phases of the eclipse (cf. Āryabhaṭīya, Gola 46, and Brāhmasphuṭasiddhānta 4,19).

[1]) Weidner, Archiv für Orientforschung 17 (1954) p. 71 ff.

VI,11. The direction of impact is determined by this rule. Four cases are to be considered (cf. Fig. 28), depending on the position of the Moon relative to the node and the character of the node (ascending or descending). Assuming that the Moon is ahead of the ascending node, thus $\lambda_c - \lambda_m > 0$, then the impact (i.e., the point of contact with the shadow) occurs on the upper (northern) half of the Moon; conversely,

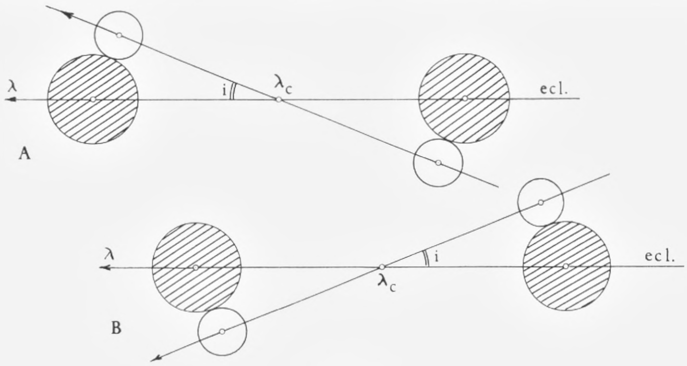

Fig. 28.

when the Moon has passed the node, thus $360 - \lambda_c + \lambda_m = \lambda_m - \lambda_c > 0$, the impact is on the lower (southern) half. The situation is reversed at the descending node.

"The beginning of Aries" ($\lambda = 0°$) seems to mean the ascending node, "the end of Virgo" ($\lambda = 180°$) the descending node. The number of synodic months in an 18-year eclipse cycle is 223, but the role of this number in the present context remains obscure to us.

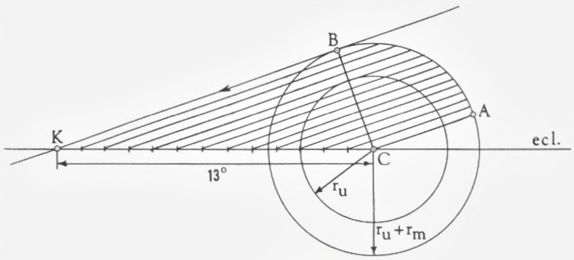

Fig. 29.

VI,12–15. In VI,12 we are given explicitly the apparent diameters

$$r_m = 0;17° \qquad r_u = 0;38° \tag{1}$$

already used in VI,3 and VI,5. For the graphic representation of an eclipse concentric circles have to be drawn of radius r_u, r_m, $r_u + r_m$ (cf. Fig. 30).

The 13 parallel lines mentioned in VI,13 are most likely related to the 13° given in VI,2 as eclipse limits. If this is the case the parallel lines would be drawn parallel to the lunar orbit, subdividing the radius $CB = r_u + r_m$ (cf. Fig. 29) into 13 equal intervals which would be a measure of the eclipse magnitudes between zero and maximum duration as function of β.

This construction can now be related to VI,7 where a 13-division of a quadrant of the Moon's circumference is mentioned, such that one can read off the distance of the point of first (and last) contact with respect to the line EW (cf. Fig. 30). We construct the point H' of the first contact when the moon is at G, where FG is one of the 13 parallels from Fig. 29. According to VI,12 one has at the common midpoint C also a circle of radius r_{m}. Hence H at this fixed circle shows the same position with respect to the Moon's center as H' with respect to G. For all lunar positions on the

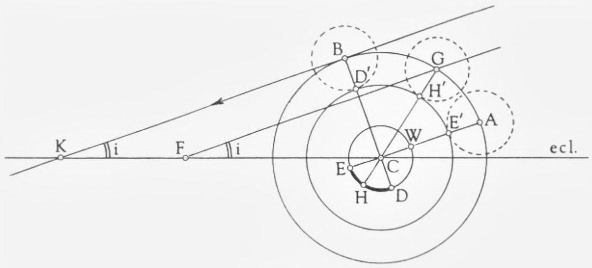

Fig. 30.

quadrant between no eclipse at B and greatest duration at A we thus obtain on the quadrant ED of the Moon's image with center C the images of the points of first contact. And since we have 13 parallels FG we have also 13 sections (of unequal length) on the quadrant ED.

The verse VI,15 gives obvious rules for the sides at which first contact will be seen at lunar- and solar-eclipses. The middle of the eclipse coincides with the syzygy, i.e., with the end of a tithi.

The "projection" of an eclipse (VI,14) is discussed later in XI,1–5. It consists of a graphical method by means of which the phases of an eclipse can be illustrated with respect to the eclipsed body which is represented by a fixed circle in the center of the diagram. Our present chapter refrains from giving specific rules.

Chapter VII

VII,1. This verse (which is repeated in VIII,9) gives a rule for the approximate determination of the longitudinal components p_λ of the (difference between) lunar (and solar) parallax, expressed in nāḍīs, i.e., essentially the time interval between true and apparent conjunction. It is assumed that $p_\lambda = 0$ in the meridian (here, as is often the case, substituted for the nonagesimal) and that the maximum $p_{\lambda\,\max}$ of p_λ occurs at an hour angle of $\pm 90°$, meant to represent a position in the horizon. This "horizontal parallax" $p_{\lambda\,\max} = p_0$ is estimated as 4 nāḍīs. A delay of this amount between true and apparent conjunction represents a reasonable estimate for the horizontal (relative) parallax since $0;4^{d}\cdot 12;10^{o/d} \approx 0;49°$ is the corresponding longitudinal displacement.

Finally it is assumed that p_λ varies sinusoidally between the value of 4^{n} in the horizon and 0 at the meridian, hence (reckoned in nāḍīs)

$$p_\lambda = 4 \sin \Delta t° = \frac{\mathrm{Sin}\,\Delta t°}{30} = \frac{\mathrm{Sin}(6\Delta t^{n})}{30} \tag{1}$$

where $\Delta t°$ is reckoned in degrees, Δt^{n} in nāḍīs.

VII,2. This verse is corrupt, but seems to contain the elements necessary for the computation of the latitudinal parallax. The translation, then, here more than elsewhere, depends on astronomy rather than grammar. Cf. also VIII,10–14.

The computation of the latitudinal component p_β of the parallax is based on the relation

$$p_\beta = p_0 \sin z \tag{2}$$

where p_0 is the horizontal parallax, z the zenith distance of the nonagesimal; p_β is now measured in degrees and has a constant value along the ecliptic (or, in practice, along the lunar orbit) for a fixed zenith distance. The latter is found from $\varphi \pm \delta \pm \beta$ with proper rules for the signs of the declination δ and the latitude β.

If one assumes for the horizontal parallax the same estimate of 4 nāḍīs as in the preceding verse, and $13^{o/d}$ for the lunar velocity[1]) one has

$$p_0 \approx 0;4^{d}\cdot 13^{o/d} = 0;52°. \tag{3}$$

The text obtains the same result by saying

$$p_0 = 4\cdot\tfrac{5}{23} \approx 4\cdot 0;13 = 0;52 \tag{3a}$$

or finally, by introducing nāḍīs:

$$p_\beta = 4^{n}\frac{5}{23}\cdot\frac{\mathrm{Sin}\,z}{2} \approx 0;4^{d}\cdot 0;13\frac{\mathrm{Sin}\,z}{2} = 4\cdot 0;13\frac{\mathrm{Sin}\,z}{120} = p_0 \sin z \tag{4}$$

as it should be according to (2).

VII,3. Here we find rules for the signs of the corrections due to parallax. The text as it stands seems to have confused the different cases. In fact the longitudinal component is subtractive to the west of the nonagesimal, additive to the east. The latitudinal component, however, is always negative.

VII,4. Let Δt be the time (of the syzygy?) after sunrise or before sunset, reckoned in nāḍīs, δ_m the declination of the Moon. The text seems to instruct us to compute $\Delta t\cdot\delta_m/80$ (or, perhaps, $\mathrm{Sin}\,\Delta t\cdot\delta_m/80$).

[1]) Actually one should take also here the relative velocity, i.e., $\approx 12^{o/d}$.

At any rate this quotient is zero at the horizon ($\Delta t = 0$) and this seems to exclude any interpretation as parallax. We have no explanation to offer.

VII,5. As in VI,2 one introduces a modified nodal position

$$\lambda_c' = \lambda_c + 0;36^\circ \tag{1}$$

λ_c being the longitude of the node. Then the limits for lunar eclipses are

$$|\lambda_m - \lambda_c'| < 13^\circ \tag{2}$$

(as in VI,2) and

$$|\lambda_m - \lambda_c'| < 8^\circ \tag{3}$$

for solar eclipses, λ_m being the longitude of the Moon.

If we assume, as in VI,3, $r_m = 0;17^\circ$, $i = 4^\circ$, one has

$$\sin i \approx 0;4,11$$

and thus with (3)

$$r_s = 0;4,11 \cdot 8 - 0;17 = 0;16,28$$

hence probably originally

$$r_s = r_m = 0;17^\circ \tag{4}$$

whereas the Romakasiddhānta assumes (cf. VIII,13) the slightly different values

$$r_s = 0;15^\circ \quad r_m = 0;17^\circ \tag{5}$$

for the apparent radii of the luminaries.

VII,6. The duration of an eclipse is given by

$$\tfrac{3}{4}\sqrt{\Delta^2 - \Delta\lambda^2} \tag{1}$$

where, according to VII,5

$$\Delta = \begin{cases} 13^\circ & \text{for lunar eclipses} \\ 8^\circ & \text{for solar eclipses.} \end{cases} \tag{2}$$

In order to check this rule we use again Fig. 24 (p. 52) calling CK $= \Delta$, CF $= \Delta\lambda$ and

$$r = \begin{cases} \mathrm{r_u} + r_m & \text{for lunar eclipses} \\ \mathrm{r_s} + r_m & \text{for solar eclipses.} \end{cases}$$

Then $r = \Delta \sin i$ and $\beta = \Delta\lambda \sin i$ and thus the distance AB which measures the half duration

$$\mathrm{AB} = \sin i \sqrt{\Delta^2 - \Delta\lambda^2} \approx 0;4\sqrt{\Delta^2 - \Delta\lambda^2} \tag{3}$$

The distance AB is here measured in degrees.

Assuming as the daily increase of elongation 12° we obtain for the total duration of the eclipse

$$t = \frac{0;4}{12}\sqrt{\Delta^2 - \Delta\lambda^2} = 0;0,40^{\mathrm{d}}\sqrt{\Delta^2 - \Delta\lambda^2}$$

$$= \frac{2^{\mathrm{n}}}{3}\sqrt{\Delta^2 - \Delta\lambda^2}$$

instead of (1), reckoned in nāḍīs. We cannot explain the origin of this discrepancy.

Chapter VIII

VIII,1. The mean longitude $\bar{\lambda}$ of the Sun, reckoned in rotations, is obtained from the ahargaṇa a by means of

$$\bar{\lambda} = \frac{a \cdot 150 - 65}{54\,787} = \frac{a \cdot 2{,}30 - 1{,}5}{15{,}13{,}7}\ \text{rot.} \tag{1}$$

Since

$$\frac{2{,}30}{15{,}13{,}7} = \frac{1{,}0{,}0}{6{,}5{,}14{,}48}$$

one sees that (1) is based on the assumption that

$$1\ \text{year} = 365;14{,}48^{\mathrm{d}} \tag{2}$$

i.e. on the value adopted for the tropical year by Hipparchus and Ptolemy; cf. also I,15 (1).

For the constant $-65/54\,787$ cf. below at VIII,5.

VIII,2–3. The solar apogee is placed at ♊ 15°. If α denotes the anomaly reckoned from this point the following equations of center θ are assumed (cf. Fig. 31):

α	$\Delta\theta$	θ
0°		0
15	0;20 + 0;15 − 0;0,18 = 0;34,42°	0;34,42°
30	+ 0;14 − 0;0, 5	1; 8,37
45	+ 0;10 + 0;0, 2	1;38,39
60	+ 0; 4 + 0;0,10	2; 2,49
75	− 0; 6 + 0;0,16	2;17, 5
90	− 0;14 + 0;0,18	2;23,23

The maximum equation of 2;23,23 would correspond, for $R = 60$, to an eccentricity of 2;17. Cf. also IX,7–8.

VIII,4. The sidereal mean longitude of the Moon, reckoned in rotations, is given by

$$\bar{\lambda} = \frac{a \cdot 38\,100 - 1984}{1\,040\,953} = \frac{a \cdot 10{,}35{,}0 - 33{,}4}{4{,}49{,}9{,}13} \text{ rot.} \tag{1}$$

From

$$\frac{4{,}49{,}9{,}13}{10{,}35{,}0} \approx 27;19{,}17{,}45{,}50, \ldots^{d} \tag{2}$$

one knows the length of the sidereal month. In I,15 (5) we have shown that (2) is based on the relation

$$4{,}14 \text{ sid.m.} = 3{,}55 \text{ syn.m.} = 19^{y}. \tag{3}$$

For the epoch constant (kṣepa), obtained for $\alpha = 0$, cf. the commentary on VIII,5.

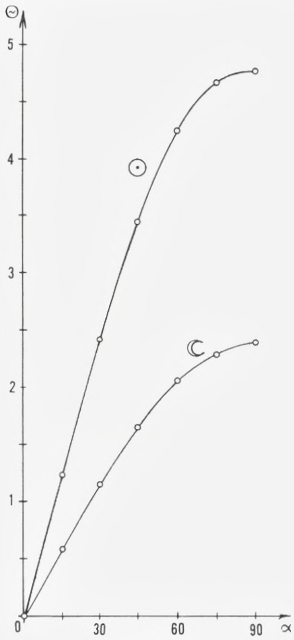

Fig 31.

VIII,5. The longitude of the lunar anomaly is give by

$$\bar{\alpha} = \frac{a \cdot 110 + 609}{3031} = \frac{a \cdot 1{,}50 + 10{,}9}{50{,}31}. \tag{1}$$

This rule is based on an anomalistic month of

$$\frac{50{,}31}{1{,}50} \approx 27;33{,}16{,}21{,}49, \ldots^{d} \tag{2}$$

Cf. for this relation above II,2–6.

The epoch constants in VIII,1,4, and 5 indicate that the initial positions for the Sun, the Moon, and the lunar anomaly at epoch were respectively

$$\text{for the Sun: } -\frac{1,5}{15,13,7}\text{ revol.} = -\frac{6,30,0^\circ}{15,13,7} \approx -0;25,38^\circ = 359;34,22^\circ \qquad (3)$$

$$\text{for the Moon: } -\frac{33,4}{4,49,9,13}\text{ revol.} = -\frac{3,18,24,0^\circ}{4,49,9,13} \approx -0;41,10^\circ = 359;18,50^\circ \qquad (4)$$

$$\text{for the lunar anomaly: } \frac{10,9}{50,31}\text{ revol.} = -\frac{1,0,54,0^\circ}{50,31} \approx 72;19,57^\circ. \qquad (5)$$

The longitude of the lunar apogee, λ_A, can be found (cf. Fig. 58 p. 101) from

$$\lambda_A = \bar{\lambda} - \bar{\alpha}. \qquad (6)$$

Consequently (4) and (5) give

$$\lambda_A = 359;18,50^\circ - 72;19,57^\circ = 286;58,53^\circ \qquad (7)$$

as position of the apogee at epoch.

These here determined epoch constants refer, according to VIII,1, to sunset at Avantī. In I,8 the epoch of the Romakasiddhānta and in XV,18 the epoch of Lāṭācārya is said to be sunset at Yavanapura.

VIII,6. As function of the anomaly α the following equations θ are prescribed for the motion of the Moon (cf. Fig. 31)

α	$\Delta\theta$	θ
0°		0
15	1° + 0;14 = 1;14°	1;14°
30	+ 0;11 = 1;11	2;25
45	+ 0; 2 = 1; 2	3;27
60	4·0;18 − 8·0;3 = 0;48	4;15
75	5·0;5 = 0;25	4;40
90	6·0;16 − 1;30 = 0; 6	4;46

The maximum equation of 4;46° would correspond to an eccentricity of 4;33 for $R = 60$. The above given numbers are, however, not very secure; one would prefer a maximum equation of 4;56° as found in the ārdharātrika system (cf. IX,7).

VIII,7. Crude approximations for the daily motion in longitude (13;10°) and in anomaly (13;4°). Cf. also IX,11–12.

VIII,8. For the ahargaṇa a the ascending lunar node has the longitude

$$\lambda_c = \frac{a \cdot 24 + 56\,266}{163\,111} = \frac{a \cdot 24 + 15{,}37{,}46}{45{,}18{,}31}. \tag{1}$$

Since

$$\frac{45{,}18{,}31}{24} = 1{,}53{,}16;17{,}30^{d} \tag{2}$$

we have in (2) the number of days required for one revolution of the lunar nodes (about $18^1/_2$ years). From it one finds for the retrograde daily motion

$$\frac{6{,}0}{1{,}53{,}16;17{,}30} \approx 0;3{,}10{,}41{,}32{,}\ldots{}^{\circ/d}. \tag{3}$$

The kṣepa in (1) indicates that the longitude of the ascending node at epoch was

$$-\frac{15{,}37{,}46}{45{,}18{,}31} \text{ revolutions} \approx -124;11{,}2^\circ = 235;48{,}58^\circ \tag{4}$$

in close agreement with III,29.

VIII,9. The same rule for the determination of the longitudinal parallax as given in VII,1 (q.v.).

VIII,10–14. The goal of this section is the determination of the latitudinal parallax p_β of the moon, or of the apparent latitude β_a of the Moon at a solar eclipse.

First one has to find the nonagesimal V of the ecliptic, i.e. the midpoint of the semicircle of the ecliptic that is above the horizon. Having determined for the given moment the ascendent H one has

$$\lambda(\mathrm{V}) = \lambda(\mathrm{H}) + 270^\circ = \lambda(\mathrm{H}) - 90^\circ. \tag{1}$$

Hence one can also find the declination $\delta(\mathrm{V})$ which belongs to $\lambda(\mathrm{V})$.

Let U be the point of the lunar orbit for which (cf. Fig. 32)

$$\lambda(\mathrm{U}) = \lambda(\mathrm{V}),$$

β its latitude, ω its distance from the ascending node. Then it is assumed that approximately

$$\delta(\mathrm{U}) = \delta(\mathrm{V}) \mp \beta, \tag{2}$$

i.e., the directions of latitudes and of declinations at V are identified. Since for a given moment the longitude λ_c of the ascending node can be considered to be known one can also find $\beta(\mathrm{U})$ from

$$\beta = \tfrac{2}{60}\,\mathrm{Sin}(\lambda(\mathrm{H}) - (\lambda_c + 90)). \tag{3}$$

Indeed, it follows from (1) that (with $R = 120$)

$$\beta^\circ = \tfrac{2}{60}\,\mathrm{Sin}(\lambda(V) - \lambda_c) \approx \tfrac{2}{60}\,\mathrm{Sin}\,\omega = 4\,\sin\omega. \tag{3a}$$

Accurately one should have

$$\sin\beta = \sin i\,\sin\omega$$

but for small angles

$$\sin\beta \approx \beta = \beta^\circ\frac{\pi}{180} \approx \frac{\beta^\circ}{60} \quad (\pi \approx 3)$$

hence

$$\beta^\circ \approx 60\,\sin\beta = 60\,\sin i\,\sin\omega \tag{3b}$$

thus from (3a)

$$\sin i \approx 0;4 \approx 0;4,11 \quad \text{thus } i \approx 4^\circ. \tag{3c}$$

It is assumed next that the zenith distance of U can be found from

$$z(V) = \varphi \pm \delta(U) \tag{4}$$

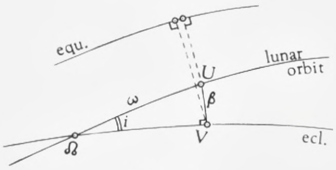

Fig. 32.

which is the same as to say that (approximately) V, U, and Midheaven lie in the same meridian. With z thus determined the latitudinal parallax is computed from

$$p_\beta = \frac{v_m\,\mathrm{Sin}\,z}{1800} \tag{5}$$

where v_m is the lunar velocity.

Since p_β can be considered constant along the ecliptic[1]) (or along the lunar orbit) with the value

$$p_\beta = p_0\,\sin z \tag{6}$$

we can obtain an estimate for the "horizontal parallax" p_0 by using (5) in the form

$$p_\beta = \frac{v_m}{15}\sin z.$$

Consequently

$$p_0 = 0;4\,v_m \approx 0;53^\circ \tag{7}$$

since $v_m \approx 13;10,35^\circ/^d$.

With p_β known from (5) one finds the apparent latitude of the Moon

$$\beta_a = \beta \pm p_\beta \tag{8}$$

[1]) Cf. Neugebauer, Al-Khwārizmī, p. 122 f.; also below IX,24–25.

where the true latitude β is found in minutes of arc from

$$\beta = \tfrac{21}{9}\,\mathrm{Sin}\,\omega. \tag{9}$$

For small β we use again (3b)

$$\beta^\circ \approx \tfrac{1}{2}\sin i\,\mathrm{Sin}\,\omega.$$

Hence from (9), since $\beta^\circ \approx 60\beta$

$$\sin i \approx \frac{2\cdot 21}{9\cdot 60} = 0;4,40$$

thus

$$i \approx 4;30^\circ \tag{10}$$

in contrast to (3c); cf. also IX,6.

VIII,15. In VIII,13 the following mean values for the apparent diameters of Sun and Moon were given

$$\bar{d}_{\mathrm{s}} = 0;30^\circ \quad \bar{d}_{\mathrm{m}} = 0;34^\circ. \tag{1}$$

For the true apparent diameters it is assumed that the ratio of diameter to velocity remains constant. Consequently

$$d_{\mathrm{s}} = \frac{\bar{d}_{\mathrm{s}}}{\bar{v}_{\mathrm{s}}}\,v_{\mathrm{s}} \qquad d_{\mathrm{m}} = \frac{\bar{d}_{\mathrm{m}}}{\bar{v}_{\mathrm{m}}}\,v_{\mathrm{m}}. \tag{2}$$

VIII,16. Let β_{a} be the apparent latitude of the moon, found in VIII,10–14 (8). Then it follows from Fig. 33 that the duration of a solar eclipse between first and last contact is given by

$$2\mathrm{AB} = \sqrt{(r_{\mathrm{s}}+r_{\mathrm{m}})^2-\beta_{\mathrm{a}}^2}. \tag{1}$$

The same rule is given in IX,26.

Note that the text here and in VIII,17 uses avanati, i.e. p_β, in the sense of "corrected latitude" i.e. $\beta - p_\beta = \beta_{\mathrm{a}}$.

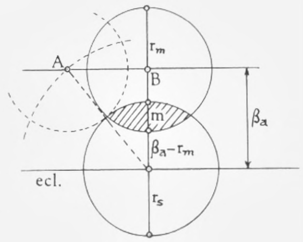

Fig. 33.

VIII,17–18. Fig. 33 shows that the obscured part of the Sun at the middle of the eclipse has the width

$$m = r_s - (\beta_a - r_m) = r_s + r_m - \beta_a. \tag{1}$$

All quantities should be measured in minutes of arc, hence also m. Nevertheless the text calls the result "digits". Cf. also X,5–6.

Chapter IX

IX,1. The mean longitude $\bar{\lambda}_s$ of the Sun, reckoned in sidereal rotations, is found for the ahargaṇa a by means of

$$\bar{\lambda}_s = \frac{a \cdot 800 - 442}{292\ 207} = \frac{a \cdot 13{,}20 - 7{,}22}{1{,}21{,}10{,}7}\ \text{rot.} \tag{1}$$

Since

$$\frac{1{,}21{,}10{,}7}{13{,}20} = 6{,}5;15{,}31{,}30 \tag{2}$$

we see that (1) is based on a sidereal year of $365;15{,}31{,}30^d$ (exactly).

The daily mean motion of the Sun amounts accordingly to

$$\frac{6{,}0}{6{,}5;15{,}31{,}30} \approx 0;59{,}8{,}10{,}10{,}37, \ldots^{\circ/d}. \tag{3}$$

The exact equivalent of (2) is the statement of the ārdharātrika system (cf. I,14) that

$$4\ 320\ 000\ (= 20{,}0{,}0{,}0)\ \text{sid. years} = 1\ 577\ 917\ 800\ (= 2{,}1{,}45{,}10{,}30{,}0)\ \text{days} \tag{4}$$

as is evident in the sexagesimal version since the number of days in (4), multiplied by a factor 3 gives the right hand side in (2).

For the epoch-correction (kṣepa)

$$-\frac{7{,}22 \cdot 6{,}0^\circ}{1{,}21{,}10{,}7} \approx -0;32{,}40^\circ \tag{5}$$

cf. the commentary to XVI,1–9.

IX,2. The mean longitude of the Moon, reckoned in sidereal rotations, is obtained from

$$\bar{\lambda}_m = \frac{a \cdot 900\ 000 - 670\ 217}{24\ 589\ 506} = \frac{a \cdot 4{,}10{,}0{,}0 - 3{,}6{,}10{,}17}{1{,}53{,}50{,}25{,}6}\ \text{rot.} \tag{1}$$

The quotient

$$\frac{1{,}53{,}50{,}25{,}6}{4{,}10{,}0{,}0} = 27;19{,}18{,}1{,}26{,}24\ \text{days} \tag{2}$$

gives the length of the sidereal month (exactly). From (2) one finds for the daily mean motion of the Moon

$$\frac{4,10,0,0 \cdot 6,0}{1,53,50,25,6} = \frac{25}{1;53,50,25,6} = 13;10,34,52,9\ldots{}^{\circ/d}. \qquad (3)$$

For the epoch correction

$$-\frac{3,6,10,17 \cdot 6,0^{\circ}}{1,53,50,25,6} \approx -9;48,44^{\circ} \qquad (4)$$

cf. the commentary to XVI,1–9.

IX,3. The number of sidereal rotations of the Moon's apogee at the ahargaṇa a is given by

$$\frac{a \cdot 900 + 2\ 260\ 356}{2\ 908\ 789} = \frac{a \cdot 15,0 + 10,27,52,36}{13,27,59,49}\ \text{rot.} \qquad (1)$$

Consequently one rotation takes

$$\frac{13,27,59,49}{15,0} = 53,51;59,16\ \text{days} \qquad (2)$$

(exactly). The daily motion of the Moon's apsidal line amounts therefore to

$$\frac{6,0}{53,51;59,16} \approx 0;6,40,59,29,\ldots{}^{\circ/d}. \qquad (3)$$

rounded to $0;6,40 = 1/9^{\circ/d}$ in IX,12 (3).

For the epoch-correction

$$\frac{10,27,52,36 \cdot 6,0^{\circ}}{13,27,59,49} \approx 279;44,53^{\circ} \qquad (4)$$

cf. the commentary to XVI,1–9.

IX,4. It is of interest to note that the basic intervals which correspond to the rules formulated in IX,1–3 (and IX,5) are all expressed exactly by finite sexagesimal fractions, no doubt intentionally. In IX,4 small corrections of the lunar parameters are introduced the purpose of which is an adjustment to the parameters of the ārdharātrika system which in turn is based on simple sexagesimal relations without approximations but now referring to the huge interval of 20,0,0,0 years. The corrections in IX,4 serve to bridge the inevitable gap between the exact identities of the individual basic periods and similar identities for the cosmic intervals.

The rules given in IX,4 are as follows:

For each sidereal revolution of the mean Moon $\bar{\lambda}_m$ should be modified by

$$\left.\begin{array}{l} -\frac{51}{3120} = -\frac{51}{52,0}\text{ seconds} \approx -0;0,0,0,58,50,46,\ldots^{\circ} \end{array}\right\} \quad (1)$$

and

For each sidereal revolution of the Moon's apogee one should add

$$\frac{10}{297} = \frac{10}{4,57}\text{ seconds} \approx 0;0,0,2,1,\ldots^{\circ} \quad (2)$$

In order to relate these rules to the ārdharātrika system we proceed as follows. First we combine IX,1 (2)

$$1 \text{ year} = 6,5;15,31,30^{d} = \frac{1,21,10,7}{16\cdot 50}$$

with IX,2 (2)

$$1 \text{ sid. month} = 27;19,18,1,26,24^{d} = \frac{1,53,50;25,6}{5\cdot 50}$$

and thus find

$$\left.\begin{array}{l} 1\text{ year} = \frac{1,21,10,7\cdot 5}{1,53,50;25,6\cdot 16} = \frac{6,45,50,35}{30,21,26;41,36} = \\ 13;22,7,46,50,11,5,24,\ldots \text{ sid. months.} \end{array}\right\} \quad (3)$$

In the ārdharātrika system, however, it is assumed that

$$4\ 320\ 000 \ (= 20,0,0,0) \text{ sid. y.} = 57\ 753\ 336 \ (= 4,27,22,35,36) \text{ sid. m.} \quad (4a)$$

Multiplication by the factor 3 gives this assumption the form

$$1 \text{ year} = 13;22,7,46,48 \text{ sid. months.} \quad (4b)$$

Comparison of (4b) with (3) shows that the rule of IX,2 produces a result which is 0;0,0,0,2,11,5,24 sidereal months in excess over the result obtained by the ārdharātrika norm. Since each sidereal month corresponds to a complete rotation the above found excess amounts to

$$0;0,0,0,2,11,5,24\cdot 6,0^{\circ} = 0;0,0,13,6,32,24^{\circ}$$

per year, i.e., because of (4b)

$$\frac{0;13,6,32,24 \text{ seconds}}{13;22,7,46,48} \approx \frac{51}{52,0}\text{ seconds}^{1)}$$

as subtractive correction. This is the above given rule (1).

[1]) Indeed $13,6,32,24\cdot 52 \approx 11,21,40$ and $13,22,7,46,48\cdot 51 \approx 11,21,48$.

Similarly IX,3 (2) in combination with IX,1 (2) shows that

$$1 \text{ year} = \frac{1,21,10,7 \cdot 9}{13,27,59,49 \cdot 8} = \frac{12,10,31,3}{1,47,43,58,32} = 0;6,46,50,56,57,43,23, \ldots \text{ rot.} \qquad (5)$$

of the lunar apogee. The ārdharātrika system, however, postulates

$$4\,320\,000 \ (= 20,0,0,0) \text{ years} = 488\,219 \ (= 2,15,36,59) \text{ rotations} \qquad (6a)$$

or the equivalent

$$1 \text{ year} = 0;6,46,50,57 \text{ rotations} \qquad (6b)$$

i.e. 0;0,0,0,0,2,16,37 rotations = 0;0,0,2,16,37 · 6,0 seconds = 0;0,13,39,42 seconds more than in (5) in each year. Since, according to (5) each year contains 0;6,46,... rotations the correction per rotation amounts to

$$\frac{0;13,39,42}{6;46,50,57} \approx \frac{10}{4,57} \text{ seconds}^{1)}$$

as required in (2).

No correction is needed for the mean Sun since IX,1 (2) and IX,1 (4) are exactly equivalent, both being based on ārdharātrika parameters.

IX,5. The longitude of the ascending node of the Moon at the ahargaṇa a, reckoned in sidereal rotations, is found from

$$\lambda_n = \frac{a \cdot 2700 + 6\,313\,219}{18\,345\,822} = \frac{a \cdot 45,0 + 29,13,40,19}{1,24,56,3,42} \text{ rotations.} \qquad (1)$$

The time required for one sidereal rotation is therefore (exactly) given by

$$\frac{1,24,56,3,42}{45,0} = 1,53,14;44,56^{d}. \qquad (2)$$

From (2) and from the number 2,1,45,10,30,0 assumed in the ārdharātrika system for the days contained in the Mahāyuga (cf. (4) in IX,1) one finds for the number of sidereal rotations of the nodes in that period

$$\frac{2,1,45,10,30,0}{1,53,14;44,56} = 1,4,30,26;3, \ldots \text{ rot.} \qquad (3)$$

hence not exactly an integer number. In fact, however, the ārdharātrika system postulates exactly 232 226 = 1,4,30,26 rotations of the nodes[2]) in a Mahāyuga. One

[1]) Indeed 13,39,42 · 4,57 ≈ 1,7,40 and 6,46,50,57 · 10 ≈ 1,7,48.
[2]) Cf. note 1 to I,14 Table 1.

expects, then, a correction to be applied to the longitude of the ascending lunar node such as was applied to the mean longitude and the apogee of the Moon in IX,4. Perhaps such a correction has accidentally been dropped from the text.

Again from (2) one can derive the corresponding daily motion of the nodes:

$$-\frac{6,0}{1,53,14;44,56} = -0;3,10,44,7,53,\ldots^{\circ/d}. \tag{4}$$

Cf. also III,28–29.

The epoch correction contained in (1) has to be reckoned negative since the rotation of the nodes proceeds in retrograde direction. Converted to degrees it amounts to

$$-\frac{29,13,40,19\cdot 6,0^\circ}{1,24,56,3,42} \approx -123;53,3 = 236;6,57^\circ. \tag{5}$$

Cf. for it the commentary to XVI,1–9 (p. 100).

IX,6. The value

$$i \approx 4;30^\circ$$

for the inclination of the lunar orbit agrees with VIII,10–14 (10).

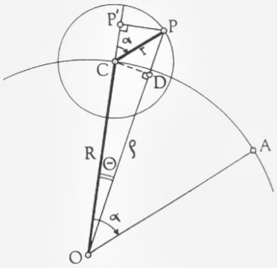

Fig. 34.

IX,7–8. Here we have rules for the determination of the equation of center for Sun and Moon as function of the anomaly α, based on a simple epicyclic model (cf. Fig. 34). The anomaly α is found from

$$\alpha = \bar{\lambda} - \lambda_A \tag{1}$$

$\bar{\lambda}$ being the mean longitude at the given moment, λ_A the longitude of the apogee.

For the Sun, in the ārdharātrika system, one has always

$$\lambda_A = 80^\circ \tag{2}$$

as fixed sidereal longitude. For the Moon λ_A has to be found with IX,3 and IX,4.

Let c be the length of the circumference of the epicycle, measured in the same units (misleadingly called "degrees") in which the circumference of the deferent measures 360. Then it is assumed that

$$c = \begin{cases} 14 \text{ for the Sun} \\ 31 \text{ for the Moon.} \end{cases} \tag{3}$$

Using c and α from (3) and (1) respectively the equation θ is found from

$$\operatorname{Sin}\theta = \frac{c \operatorname{Sin}\alpha}{360}. \tag{4}$$

It is easy to see that (4) is the result of a plausible approximation (cf. Fig. 34). Obviously

$$\frac{c}{360} = \frac{r}{R} = \frac{r \sin\alpha}{R \sin\alpha} = \frac{\mathrm{PP}'}{\operatorname{Sin}\alpha}.$$

Since r is small in comparison to R[1]) we may assume that $\mathrm{PP}' \approx \mathrm{CD}$, hence

$$\frac{c}{360} \approx \frac{\mathrm{CD}}{\operatorname{Sin}\alpha} = \frac{R \sin\theta}{\operatorname{Sin}\alpha} = \frac{\operatorname{Sin}\theta}{\operatorname{Sin}\alpha}$$

which proves (4).

It follows from (3) and (4) that

$$r = \begin{cases} 0;2,20 \text{ for the Sun} \\ 0;5,10 \text{ for the Moon} \end{cases} \tag{5}$$

which leads to a maximum equation of

$$\theta_{\max} = \begin{cases} 2;14^\circ \text{ for the Sun} \\ 4;56^\circ \text{ for the Moon.} \end{cases} \tag{6}$$

Cf. also VIII,2–3 and VIII,6. In terms of Greek astronomy the eccentricities corresponding to (5) would be 2;20 and 5;10 respectively ($R = 60$); cf. also XVI,12–14 Table 22 (p. 102).

IX,9. This is a rule for finding the daily increase in the solar and lunar equations. Since the increase of both θ and α in a day will be very small we replace (3) in IX,7–8 by $\theta = c\alpha/360$ and thus obtain

$$\theta' = \frac{c}{360}\alpha'$$

where θ' and α' denote the daily increments of θ and α respectively. For the Sun the daily increase in anomaly is the same as its daily increase v_S in longitude, reckoned in degrees per day. If b denotes the same velocity (bhukti) expressed in minutes we have therefore

[1]) Fig. 34 is not drawn to scale; for the Moon r should be about 1/12 of R, for the Sun only 1/30 of R (cf. (2)).

$$\theta' = \frac{c}{21\,600}\, b = \frac{cb}{6{,}0{,}0}.$$

For the Moon one should use the bhukti of the anomaly given in IX,12; cf. also IX,13.

IX,10. For 53;20 yojanas of distance (measured on the terrestrial equator) a time correction of ± 1 nāḍī is to be applied. With this time correction one can compute the correction to the longitudes of the planets due to the fact that one's locality is a known number of yojanas (on the equator) to the east or west of the prime meridian.

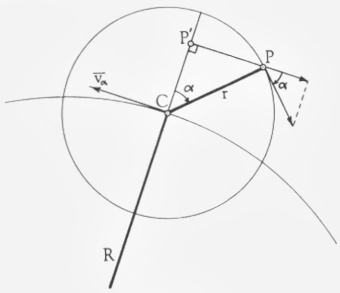

Fig. 35.

It follows from the above given numbers that

$$1^{\mathrm{d}} = 53{,}20 \ \text{yojanas} = 3200 \ \text{yojanas}$$

on the equator; cf. also III,14 and XIII,15–19. The radius of the earth is therefore about 510 yojanas.

IX,11–14. The mean motions are (cf. also VIII,7)

$$\bar{v} = \begin{cases} 0;59{,}8^{\circ}/^{\mathrm{d}} & \text{for the Sun} \\ 13;10{,}34 & \text{for the Moon.} \end{cases} \tag{1}$$

The true motion of the Moon is given by

$$v = \bar{v} \pm \bar{v}_\alpha \frac{\Delta \operatorname{Sin} \alpha}{225} \cdot \frac{c}{360} \tag{2}$$

where

$$\bar{v}_\alpha = \bar{v} - \bar{v}_{\mathrm{A}} \tag{3}$$

with

$$\bar{v}_{\mathrm{A}} \approx 0;6{,}40^{\circ/\mathrm{d}} \tag{4}$$

(cf. also IX,3) and c being defined as in IX,7–8. As shown in Fig. 35 $\bar{v}_\alpha$ represents therefore the velocity of the center C of the epicycle with respect to the apsidal line.

Since the function Sin α is tabulated in steps of 3;45° = 225′ (cf. IV,6–15, Table 8) we see that Δ Sin $\alpha/225$ is approximately the derivative of the Sine-function. Thus we have

$$\bar{v}_\alpha \frac{\Delta \operatorname{Sin} \alpha}{225} = \bar{v}_\alpha \frac{\Delta \operatorname{Sin} \alpha}{\Delta \alpha} \approx \bar{v}_\alpha \cos \alpha.$$

Because of the "reducing" factor $c/360 = r/R$ we have in $\bar{v}_\alpha c/360$ the velocity of the Moon P on the circumference of the epicycle. Consequently the multiplication with $\cos \alpha$ gives the component in the direction of $\bar{v}_\alpha$ representing (algebraically) the effect of the motion of P on the epicycle, while C moves with the constant mean velocity $\bar{v}$.

The instantaneous velocity v of P with respect to O (cf. Fig. 34) can also be described by means of the "true hypotenuse" ϱ = OP in relation to R:

$$\bar{v}/v = R/\varrho. \tag{5}$$

IX,15–16. It is the purpose of these verses to introduce the actual geocentric distances (kakṣā) of Sun and Moon instead of operating uniformly with the norm $R = 120$ for the radii of the deferents. If ϱ_s and ϱ_m represent the true hypotenuse of Sun and Moon respectively the kakṣā is defined by

$$k_s = \frac{5347}{40} \varrho_s = \frac{1,29,7}{40} \varrho_s \quad k_m = 10 \varrho_m. \tag{1}$$

By means of two more parameters

$$b_s = 517\,080 \ (= 2,23,38,0) \quad b_m = 38\,640 \ (= 10,44,0) \tag{2}$$

are found the apparent diameters of Sun and Moon

$$d_s = \frac{b_s}{k_s} = 1,4,28;11,\ldots \cdot \frac{1}{\varrho_s} \qquad d_m = \frac{b_m}{k_m} = 1,4,24 \frac{1}{\varrho_m}. \tag{3}$$

For the mean distances of the luminaries, i.e. for

$$\varrho_s = \varrho_m = R = 120$$

one obtains from (1) the geocentric distances

$$k_s = 4,27,21 \quad k_m = 20,0 \tag{4}$$

and from (3) the apparent diameters

$$d_s = 32;14,6,\ldots \quad d_m = 32;12 \tag{5}$$

obviously measured in minutes of arc.

One can now determine the absolute dimensions of the diameters $2s$ and $2m$ of Sun and Moon respectively, measured in the units of k_s and k_m. Dividing d_s and d_m by the ratio

$$3{,}0{,}0'/\pi = 10\,800'/\pi \approx 3438' = 57{,}18'$$

one changes minutes of arc to radians. Multiplication by k_s and k_m then produces $2s$ and $2m$:

$$\left.\begin{aligned} 2s &= \frac{d_s}{57{,}18} k_s = \frac{b_s}{57{,}18} = 2{,}30;24{,}5,\ldots \\ 2m &= \frac{d_m}{57{,}18} k_m = \frac{b_m}{57{,}18} = 11;14{,}20,\ldots \end{aligned}\right\} \quad (6)$$

i.e. the radii

$$s \approx 1{,}15;12 \quad m \approx 5;37 \quad (7)$$

respectively.

In IX,19–23 we shall find (cf. below p. 76) that the determination of the lunar parallax is based on the assumption that

$$e = 18 \quad (8)$$

is the radius of the earth, measured in otherwise unknown units. If we assume that these units are the same as in s and m, hence also as in k_s and k_m, we can convert all these quantities to earth radii; from (4) one obtains:

$$\left.\begin{aligned} k_s &= \frac{4{,}27{,}21}{18} e = 14{,}51;10e = 891^1/_6\, e \\ k_m &= \frac{20{,}0}{18} e = 1{,}6;40\, e = 66^2/_3\, e. \end{aligned}\right\} \quad (9)$$

The value for the lunar distance is essentially correct and in general agreement with the results obtained in ancient astronomy. This confirms the uniformity of all units of distance.

Finally one obtains the absolute sizes for Sun and Moon from (7):

$$s \approx \frac{1{,}15;12}{18} e = 4;10{,}40\, e \quad m \approx \frac{5;37}{18} e = 0;18{,}43{,}20\, e. \quad (10)$$

Hence, the radius of the Sun is about four times the radius of the earth, the radius of the Moon about one third.

IX,17–18. We are given the time difference $\Delta\alpha$ between the moment of conjunction and noon, i.e. the arc $\Sigma'C$ in Fig. 36. The corresponding ecliptic arc $\Delta\lambda = \Sigma M$ can be found from a table of right ascensions since $\Delta\alpha$ is the rising time of $\Delta\lambda$ at sphaera recta.

Since we know for the given moment the longitude λ of the conjunction we have in $\lambda + \Delta\lambda$ the longitude of M. Hence one can find from a table the declination δ_{M} of M and finally the zenith distance of M:

$$z_{\mathrm{M}} = \varphi \mp \delta_{\mathrm{M}}. \tag{1}$$

IX,19–23. These verses concern the determination of the longitudinal parallax of sun and moon at a conjunction, i.e. for a solar eclipse. The procedure is best broken up in single steps for which we give the proofs as we go along.

Let M be the culminating point of the ecliptic (cf. Fig. 37), V the nonagesimal, Z the zenith, λ_{M} and δ_{M} longitude and declination of M respectively, η the azimuth

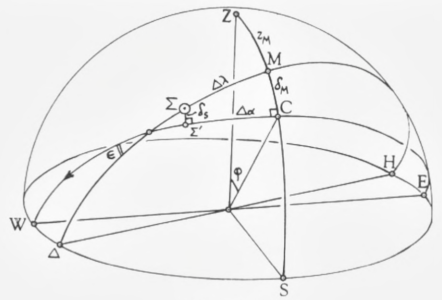

Fig. 36.

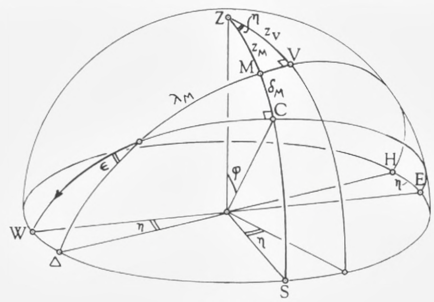

Fig. 37.

of the ascendent H, ε the obliquity of the ecliptic, φ the geographical latitude. Then, with λ_{M} known from IX,17

$$\operatorname{Sin}\eta = \frac{\operatorname{Sin}\varepsilon \cdot \operatorname{Sin}\lambda_{\mathrm{M}}}{\operatorname{Sin}\bar{\varphi}}. \tag{1}$$

Proof: $\sin\varphi = \operatorname{Sin}\delta_{\mathrm{M}}/\operatorname{Sin}\eta$ (cf. IV,39–40 (2)) and $\sin\varepsilon = \operatorname{Sin}\delta_{\mathrm{M}}/\operatorname{Sin}\lambda_{\mathrm{M}}$ (cf. IV,35–36 (2)); q.e.d.

Let z_{M} and z_{V} represent the zenith distances of M and V respectively. Then

$$\operatorname{Sin}\mathrm{VM} = \operatorname{Sin}\eta \cdot \operatorname{Sin} z_{\mathrm{M}}/R \tag{2}$$

where $\operatorname{Sin}\eta$ is known from (1) and z_{M} from IX,17–18 (1).

Proof: In the right spherical triangle ZVM holds

$$\sin\eta = \sin \mathrm{VM}/\sin z_{\mathrm{M}}$$

q.e.d.

The next step implies a very crude approximation because it assumes that the right spherical triangle ZVM can be considered as a plane triangle. Hence

$$\operatorname{Sin} z_{\mathrm{V}} = \sqrt{\mathrm{Sid}^2 z_{\mathrm{M}} - \operatorname{Sin}^2 \mathrm{VM}} \tag{3}$$

can be computed, using z_{M} from IX,17–18 (1) and VM from (2).

With z_V known from (3) one has for the altitude a_V of V

$$a_V + z_V = 90^\circ \tag{4a}$$

thus

$$\mathrm{Sin}\, a_V = \sqrt{R^2 - \mathrm{Sin}^2 z_V}\,. \tag{4b}$$

Let a_s be the altitude, z_s the zenith distance of the Sun for a given moment when the Sun has a longitudinal distance $\Delta\lambda$ ($< 90^\circ$) from the rising or setting point of the ecliptic. Then

$$\mathrm{Sin}\, a_s = \mathrm{Sin}\, \Delta\lambda \cdot \mathrm{Sin} a_V / R. \tag{5}$$

Since

$$a_s + z_s = 90^\circ \tag{6a}$$

one has

$$\mathrm{Sin}\, z_s = \sqrt{R^2 - \mathrm{Sin}^2 a_s}\,. \tag{6b}$$

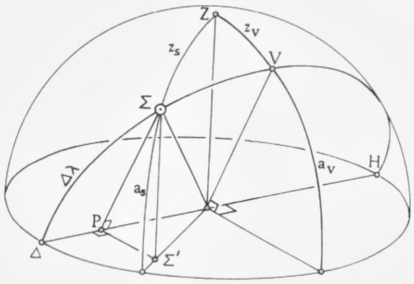

Fig. 38.

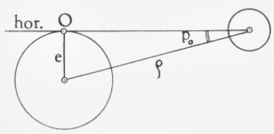

Fig. 39.

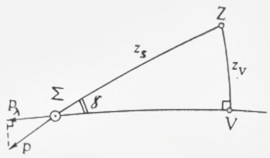

Fig. 40.

Proof of (5): it follows from Fig. 38

$$\Sigma\Sigma' = R \sin a_s = \mathrm{P}\Sigma \sin a_V$$

and

$$\mathrm{P}\Sigma = R \sin \Delta\lambda$$

q.e.d.

We have now all elements that are required for the computation of the longitudinal parallax

$$p_\lambda = \frac{18}{\varrho R}\sqrt{\mathrm{Sin}^2 z_s - \mathrm{Sin}^2 z_V} \tag{7}$$

where ϱ is the distance of the Sun or Moon (cf. Fig. 39) from the earth. If we substitute in (7) for ϱ the value ϱ_s we obtain the longitudinal solar parallax $p_{\lambda s}$, otherwise with ϱ_m the lunar parallax $p_{\lambda m}$ and, for an eclipse, when $\lambda_s = \lambda_m$, the "adjusted" longitudinal parallax

$$p'_\lambda = p_{\lambda m} - p_{\lambda s}\,. \tag{8}$$

Proof of (7): In the right spherical triangle ZVΣ (cf. Fig. 40), Σ being the Sun, we have

$$\sin\gamma = \sin z_V / \sin z_S.$$

If p is the total parallax at Σ then

$$p_\lambda = p\cos\gamma = p\sqrt{1 - \mathrm{Sin}^2 z_V/\mathrm{Sin}^2 z_S}$$
$$= \frac{p}{\mathrm{Sin}\, z_S}\sqrt{\mathrm{Sin}^2 z_S - \mathrm{Sin}^2 z_V}.$$

Comparison with (7) shows that one should have the following relation

$$p/\mathrm{Sin}\, z_S = 18/\varrho R \tag{9}$$

or

$$p = \frac{18}{\varrho}\sin z_S = p_0 \sin z_S \tag{10}$$

with

$$p_0 = \frac{18}{\varrho}. \tag{11}$$

Since $z_S = 90°$ at the hirizon we see from (10) that p_0 is the horizontal parallax. But the lunar horizontal parallax is the angle under which the radius e of the earth appears from the moon when in the horizon. Thus (11) implies a value

$$e = 18 \tag{12}$$

for the radius of the earth, a value confirmed by its use in X,1–2.

In VII,2 we found that $p_0 \approx 0;52°$, in VIII,10–14 we arrived at the estimate $p_0 \approx 0;53°$. In order to express p_0 also here in degrees we have to replace (11) by

$$p_0 = \frac{18}{\varrho}\cdot\frac{3,0}{\pi} = \frac{54,0}{\varrho\pi}. \tag{13}$$

In IX,15–16 (1b) the distance of the Moon is assumed to be measured by

$$\varrho = k_m = 20,0.$$

If we accept for π the approximation $\pi \approx 3$ we have $\varrho\pi = 1,0,0$ and thus

$$p_0 = 0;54° \tag{14}$$

which agrees sufficiently well with the previously obtained estimates.

IX,24–25. For a given zenith distance z of the nonagesimal V the latitudinal component p_β of the parallax is assumed to be constant with the value

$$p_\beta = p_0 \sin z_V$$

where p_0 represents the horizontal parallax (cf. VIII,10–14 (6)).

IX,26. The same rule for the duration of an eclipse is found in VIII,16.

IX,27. Effect of the longitudinal component of the adjusted parallax on the duration of a solar eclipse.

Chapter X

X,1–2. Let k_s and k_m denote (as in IX,15–16) the geocentric distances of Sun and Moon respectively, s, e, and u the radii of Sun, earth, and shadow at the Moon's (mean) distance, all these quantities measured in the same units. Then the diameter $2u$ of the shadow cone at the Moon's (mean) distance is found from

$$2u = \left(36 - \frac{36k_m}{90k_s/286}\right)\frac{120}{k_m}. \tag{1}$$

A more convenient form of the rule (1) would be

$$u = 36{,}0\left(\frac{1}{k_m} - \frac{3;10{,}40}{k_s}\right). \tag{1a}$$

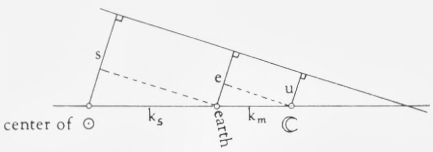

Fig. 41.

Fig. 41 illustrates the underlying argument. Obviously

$$\frac{s-e}{k_s} = \frac{e-u}{k_m} \tag{2}$$

and therefore

$$u = e\left(1 + \frac{k_m}{k_s}\right) - s\frac{k_m}{k_s}. \tag{3}$$

We now assume, as in IX,19–23 (12)

$$e = 18 \tag{4a}$$

and

$$s = 75;12 \tag{4b}$$

as derived in IX,15–16 (7). Then one obtains from (3)

$$u = 18 - 57;12\frac{k_m}{k_s} = 18\ k_m\left(\frac{1}{k_m} - \frac{3;10{,}40}{k_s}\right)$$

which agrees with (1a) if

$$k_m = 2{,}0 = R \tag{5}$$

i.e. for the normed mean distance of the Moon.

X,2–4. Let r_u and r_m be the apparent radii of shadow and Moon, both expressed in degrees, β_0 the latitude of the Moon at the moment t_0 of the true opposition, $v_m - v_s$ the relative angular velocity of the Moon with respect to the Sun, expressed in degrees per day, then

$$\Delta_1 t = \sqrt{(r_u + r_m)^2 - \beta_0^2} \cdot \frac{120}{v_m - v_s} \tag{1}$$

is in first approximation the half-duration of the eclipse (cf. Fig. 42). The underlying assumption that the latitude of the Moon at first contact is the same as at the midpoint of the eclipse, is, however, not quite correct. If one computes the latitude

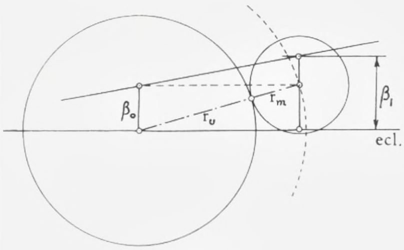

Fig. 42.

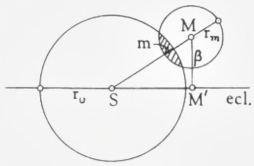

Fig. 43.

for the moment $t_0 - \Delta_1 t$ one will find a value β, slightly different from β_0. If we replace β_0 in (1) by β_1 a more accurate value $\Delta_2 t$ of the half-duration will result. Computing the latitude for $t_0 - \Delta_2 t$, etc., leads to β_2, β_3 etc. The process ends when $\Delta_{n+1} t = \Delta_n t$.

X,5–6. If t is an arbitrary moment near the moment t_0 of the true syzygy, β the corresponding latitude of the Moon, then the size m of the obscured part is found, for a lunar eclipse, from

$$m = (r_u + r_m) - \mathrm{MS} \tag{1}$$

where

$$\mathrm{MS} = \sqrt{M'\mathrm{S}^2 + \beta^2} \tag{2}$$

and

$$\mathrm{M'S} = \frac{v_m - v_s}{60}(t_0 - t). \tag{3}$$

The correctness of this rule follows immediately from Fig. 43. For a solar eclipse (1) is to be replaced by

$$m = (r_s + r_m) - \mathrm{MS}.$$

Cf. also VIII,17–18.

X,7. The half-duration of totality is computed by determining the distance AB (cf. Fig. 44) between first contact and eclipse middle:

$$AB = \sqrt{(r_u - r_m)^2 - \beta^2}.$$

The duration of totality can be found from the distance 2AB by dividing it by the relative velocity of the two luminaries, a step which is hinted at by referring to "the case of the tithi".

Chapter XI

XI,1–5. These verses concern the "projection" of eclipses, i.e., a graphical method for finding the points of first and last contact on the eclipsed body in relation to certain orthogonal diameters N′S′ and E′W′ as defined previously in VI,8 (cf. Fig. 26 p. 54).

For any given moment the direction of the ecliptic must be determined by means of an angle γ (called "deflection") with respect to the "east-west-line" E′W′ (cf. Fig.

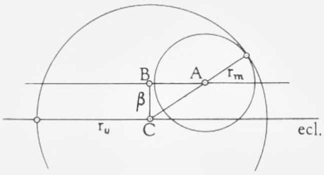

Fig. 44.

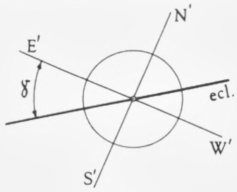

Fig. 45.

45). This angle is found from two components. The first, depending on the geographical latitude φ, is found in XI,2 from

$$\operatorname{Sin}\gamma_1 = \operatorname{Sin}\varphi \operatorname{Sin} t/R \tag{1}$$

where t is the hour angle. We have explained the reasoning underlying this rule in connection with VI,8. As was then shown the resulting angle gives actually the direction of the equator with respect to the east-west-line. One can formulate this situation also in the following form: γ_1 assumes that the eclipsed body is in the equator, i.e., $\lambda = 0°$ or $180°$. The angle of the ecliptic with the east-west-line is then $\gamma_1 \pm \varepsilon = \gamma_1 \pm \delta(\lambda + 90°)$ where $\delta(\lambda + 90°)$ is the declination at $\lambda + 90°$. For $\lambda = \pm 90°$, i.e. at the solstices, the ecliptic is parallel to the equator; then $\gamma \approx \gamma_1 = \gamma_1 + 0$, i.e. again $\gamma_1 + \delta(\lambda + 90°)$ since now $\lambda + 90° = 0°$ or $180°$, and hence $\delta = 0$. Hence, in all four cases we have found that $\gamma = \gamma_1 + \delta(\lambda + 90°)$ and XI,3 shows that this rule was considered valid not only for the equinoxes and solstices but generally:

$$\gamma = \gamma_1 + \gamma_2 = \gamma_1 \pm \delta(\lambda + 90°).$$

This, then, represents the angle between ecliptic and east-west-line at a moment when the eclipsed body has the longitude λ. Hence we can consider γ to be known for any phase of the eclipse since the times for the phases can be computed, starting from the true syzygy.

We now can turn to the graphic construction. In XI,1 we are directed to draw two concentric circles, one with radius r of the eclipsed body, the other with radius r' which is the total of the radii of eclipsed and eclipsing body, i.e.

$$\left.\begin{aligned} r &= r_{\mathrm{m}} \quad & r' &= r_{\mathrm{m}} + r_{\mathrm{u}} \quad && \text{for lunar eclipses} \\ r &= r_{\mathrm{s}} \quad & r' &= r_{\mathrm{s}} + r_{\mathrm{m}} \quad && \text{for solar eclipses.} \end{aligned}\right\} \tag{3}$$

Through the center A of these circles one draws a fixed direction which represents the ecliptic (Fig. 46) and, knowing γ from (2), one also can draw the east-west-line E′W′ through A. Perpendicular to it is N′S′ (XI,3). For each phase a diagram of this type has to be constructed because γ can vary considerably, e.g., between beginning and end of a lunar eclipse.

Suppose that Fig. 46 is constructed for the moment of first contact at a lunar eclipse. We then assume parallelism between the lunar orbit and the ecliptic. If, there-

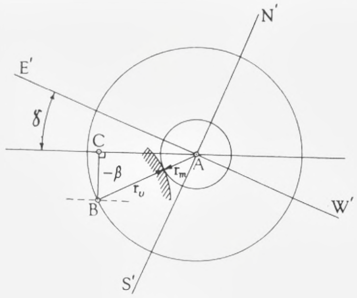

Fig. 46.

fore, A is the center of the Moon, the point B at $-\beta$ is the center of the shadow and AB intersects the rim of the Moon where the first contact will occur.

XI,6. This verse seems to deal with the optical illusion that the disk of the Moon or the Sun near the horizon seems larger than when high in the sky. An arc of one minute would then subtend (on some instrument?) $^1/_2$ digit when near the horizon but only $^1/_3$ digit at high altitude. The rule occurring here must be intended to determine the actual physical dimensions of the "projection" described in XI,1–5.

Chapter XII

XII,1. The yuga is here a cycle of 5 years, containing $12 \cdot 5 + 2 = 62$ months. It is furthermore stated after every 62 days one tithi has to be omitted, i.e.

$$62^{\tau} = 61 \text{ sid. days.} \tag{1}$$

and hence the length of one tithi

$$1^{\tau} = 61/62 \text{ sid. d.} = 0;59,1,56, \ldots \text{ sid. d.} \tag{2a}$$

and conversely

$$1 \text{ sid. d.} = 62/61 \text{ tithi} = 1;0,59,0,59, \ldots^{\tau}. \tag{2b}$$

The same relation follows from XII,3.

From (1) one obtains

$$1 \text{ yuga} = 5 \text{ years} = 62 \text{ months} = 62 \cdot 30^{\tau} = \tfrac{62}{61} \cdot 1830^{\tau} = 1830 \text{ sid. d.} \tag{3}$$

and consequently

$$1 \text{ year} = 366 \text{ sid. d.} = 365 \text{ days}. \tag{4}$$

The "years" are here the Egyptian-Persian years without intercalation.

These relationships in the Paitāmahasiddhānta of the Pañcasiddhāntikā are taken from the Jyotiṣavedāñga of Lagadha (fifth century B.C.?), and are found also in the earliest recension of the Gargasaṃhitā (first century B.C. or A.D.?). Varāhamihira's use of the technical term avama here and in I,11–16 and his statement in XII,2 that the ahargaṇa begins with sunrise indicates that he assumes that the Paitāmahasiddhānta operated with sāvana (sunrise) rather than with nākṣatra (sidereal) days. Nevertheless the only reasonable interpretation of (3)

$$5 \text{ years} = 1830 \text{ "days"}$$

is the relation (4).

XII,2. If S represents the number of completed years in the Śaka Era then

$$N = \frac{S-2}{5} \tag{1}$$

gives the number of completed yugas, i.e. the counting of the yugas begins with Śaka Era 3.

The year Śaka 3, i.e. A.D. 80, being fixed by the text as the year of epoch for Varāhamihira's Paitāmahasiddhānta, the date can be determined more closely as January 11 of this year.[1]) At this date occurred a conjunction of Sun and Moon at a tropical longitude $\lambda \approx 290°$. Consequently the Sun would enter Aries in the third month after this conjunction. Now Māgha is the second month before Caitra. Under the plausible assumption that Caitra was already in the first century the month in which the Sun enters Aries, as is the case in the fifth century, the above given date would fit all conditions of the text.

XII,3. If a is the ahargaṇa (here to be understood in sidereal days) then the number of corresponding tithis is given by

$$\tau = a(1 + \tfrac{1}{61}). \tag{1}$$

[1]) This was suggested by Kharegat [1895/7] p. 134 f. (cf. below p. 154).

The Sun traverses 27 nakṣatras in one year of 366 sidereal days (cf. XII,1 (4)), hence in a sidereal days

$$\frac{a}{366} \cdot 27 = a \cdot \frac{9}{122} \text{ nakṣatras.} \tag{2}$$

The Moon completes in 5 years of 62 months

$$62 + 5 = 67 \text{ sid. rotations} = 67 \cdot 27 \text{ nakṣatras.} \tag{3}$$

Since 5 years = 1830 sid. days (cf. XII,1(3)) the Moon progresses in a sidereal days

$$a \cdot \frac{67 \cdot 27}{1830} = a \cdot \frac{603}{610} = a\left(1 - \frac{7}{610}\right) \text{ nakṣatras.} \tag{4}$$

These rules make sense only if one is dealing with "equal" nakṣatras, i.e. with sections of uniformly 13;20° in length. If the conjunction, mentioned in XII,2, was in fact the conjunction of A.D. 80 Jan. 11 at the beginning of the nakṣatra Dhaniṣṭhā we would know that this point had in the first century a tropical longitude of about $\lambda = 290°$. Since there are 5 nakṣatras from Dhaniṣṭhā to Aśvinī, the beginning of the nakṣatra-zodiac, we find for this point $290 + 5 \cdot 13;20 = 356;40°$, i.e., Pisces 26;40°, in the first century A.D.

XII,4. The parvan is either the conjunction or the opposition of Sun and Moon. The first half of this verse, then, correctly states that the parvan is the boundary between the first and second halves (pakṣas) of a month.

The second part of this verse refers to the 17th yoga, called vyatipāta, in a series of 27 yogas. These yogas count arcs of 13;20° each, contained in the sum of solar and lunar longitude (cf. also III,20–22). According to XII,3 a conjunction occurred at epoch, in A.D. 80, at the beginning of Dhaniṣṭhā, i.e. at Capricorn 23;20° or $\lambda = 293;20°$. Consequently the yoga has the number

$$\frac{2 \cdot 293;20}{13;20} = \frac{226;40}{13;20} = 17 \tag{1}$$

i.e. vyatipāta.

Furthermore, the combined travel of Sun and Moon in a yuga of 5 years is $5 + 67 = 72$ complete rotations, i.e. $72 \cdot 360°$. But each series of 27 yogas of 13;20° each amounts also to 360°. Thus the above travel contains 72 series of yogas, accumulated during 1830 days (cf. XII,1 (3)). Hence the number of series elapsed since epoch at ahargaṇa a is given by

$$\frac{72}{1830} a = \frac{12}{305} a. \tag{2}$$

XII,5. For the interval from the winter solstice to the summer solstice (the "nothern ayana") the length of daylight C, measured in muhūrtas:

$$30 \text{ muhūrtas} = 1 \text{ day,} \tag{1}$$

is given by

$$C = \tfrac{2}{61}(n + 732) - 12 \text{ muhūrtas} \tag{2}$$

where n is the number of days since the winter solstice. For $n = 0$ this formula gives $C = 12$, for $n = \frac{366}{2} = 183$ one obtains $C = 18$. Hence (2) is based on the assumption that the length of daylight varies according to a linear zigzag function between the extrema

$$m = 12 \qquad M = 18$$

i.e. it is assumed that

$$\frac{m}{M} = \frac{2}{3} \tag{3}$$

and that the year is exactly halved by the solstices. The ratio (3) is a basic parameter in Babylonian astronomy; cf. II,8 above.

Chapter XIII

XIII,1–5, 9–14. This is the conventional cosmography of the astronomers who have adapted the Purānic (and Jaina) conception of a flat oikoumene surrounding Mount Meru to the necessities of spherical astronomy by retaining Meru as the North Pole where the gods dwell, and calling the South Pole Vaḍavāmukha, the dwelling place of the demons. The phraseology echoes verses of Lāṭadeva (see above Pt. I p. 14f.).

XIII,6–7. The "others" are Āryabhaṭa (Āryabhaṭīya, Daśagītikā 4 and Gola 9).

XIII,8. The Arhats or Jainas believe that there are two Suns, two Moons, and two sets of nakṣatras which rise alternately, so that a complete revolution of one of these bodies about the center of the flat earth, Mount Meru, takes 2 nychthemera. This system is also referred to by Brahmagupta (Brāhmasphuṭasiddhānta XI,3) who perhaps had this verse of the Pañcasiddhāntikā in mind; for a detailed description of this peculiar conception see Kirfel [1920] pp. 285–291 and plate 16.

Varāhamihira's criticism seems to imply that the Jaina theory must be false because a determined point associated with the Sun appears on two successive nychthemera. This is not a valid criticism as the Jainas could assert an absolute identity in appearance between the two opposite halves of heaven occupied by the two Suns. More cogent would have been the argument that the Jaina theory accounts for only 90° of motion of heaven between sunrise and sunset whereas observations with a gnomon would quickly establish that the motion amounts to about 180°. We do not see, however, how such an interpretation could be made of this verse.

XIII,9–10. These verses show that Laṅkā is located on the terrestrial equator, $\varphi = 0°$, and Avanti at $\varphi = \varepsilon = 24°$. Cf. also XIII,19.

XIII,15–19. The following relations are assumed for a terrestrial great circle:

$$1° = 8;53,20 \text{ yojanas} \tag{1}$$

$$1 \text{ yojana} = 0;6,45°. \tag{2}$$

Consequently one quadrant measures 800 yojanas (in agreement with III,14 and IX,10).

On the basis of the relation (1) the distance in yojanas can be found between two places on the same meridian after the geographical latitudes were determined by direct astronomical observation.

The distance from Mount Meru to Avantī (= Ujjayinī) is $586^2/_3$ yojanas, hence, according to (1), exactly 66° as it should be if $\varphi = 24°$ at Avantī (cf. XIII,10).

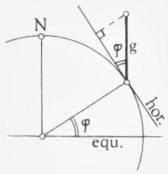

Fig. 47.

XIII,20–25. Here are consequences developed, based on the assumption made in XIII,15–19:

$$42° = 373^1/_3 \text{ yojanas north of Avantī}$$

i.e., at a latitude

$$\varphi = 66° = 90° - \varepsilon$$

the longest daylight is 60 nāḍikās = 24 hours.

Furthermore

$403^5/_9$ yojanas ≈ 45;24° north of Avantī ♐ and ♑ remain invisible
482 yoj. ≈ 54;14° north of Avantī ♏ to ♒ remain invisible
$586^2/_3$ yoj. = 66° north of Avantī ♎ to ♓ never rise, ♈ to ♍ never set.

If one schematically identifies months and zodiacal signs, the above statements imply that

at $\varphi \approx 69;24°$	longest daylight = 2 months
at $\varphi \approx 78;14°$	longest daylight = 4 months
at $\varphi = 90°$	longest daylight = 6 months.

In Almagest II,6 the same data are associated with $\varphi \approx 69;30°$, $\varphi \approx 78;20°$, $\varphi = 90°$ respectively.

XIII,26–29. Cf. the commentary to XIII,1–5, 9–14.

The astrological rules in XIII,28 are common to all Sanskrit texts on the subject since they were introduced by the Yavanajātaka of Yavaneśvara in 149/150 A.D.

XIII,30–34. The tip of a gnomon which is parallel to the earth's axis is $g \sin\varphi$ above the horizontal plane (cf. Fig. 47) at a locality of latitude φ.

XIII,35–38. On the phases of the Moon. For the pakṣas cf. the commentary on III,18–19.

XIII,39–42. The order of the planets is the familiar Greek one. From it follows the order of the rulers of the months, hours, days, and years (for which cf. I,17–21).

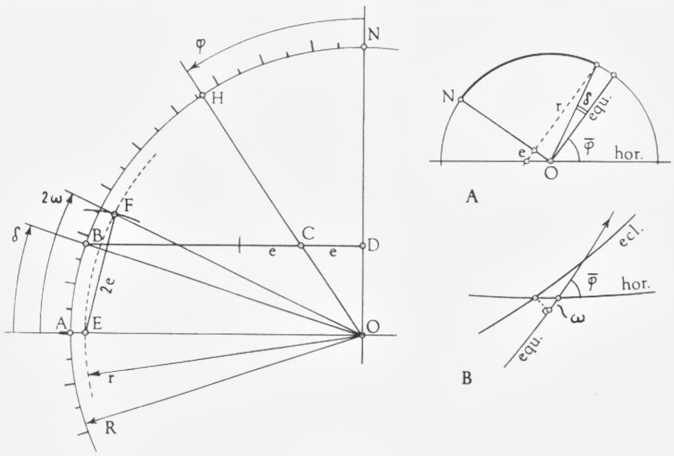

Fig. 48.

Chapter XIV

XIV,1–4. We have here a graphical method for the determination of the ascensional difference ω (cf. Fig. 48B) to given declination δ—though the formulation of the text is less general since it considers only the endpoints of zodiacal signs.[1])

The basis of construction is a graduated circle AN with center O (cf. Fig. 48), only one quadrant being actually needed. On its circumference one marks the point H such that the arc NH represents the given geographical latitude φ and AO the equator. Consequently HO represents the horizon if N is the north pole. A second arc is determined such that AB is the declination. Since AO is the equator the parallel DB is in the plane of the day-circle which belongs to δ, hence DB $= r$ the "day-radius", DC $= e$ the "earth-Sine" (cf. Fig. 48A) and

$$e = \operatorname{Sin}\delta \tan\varphi. \qquad (1)$$

[1]) The ascensional difference is called "composite" (XIV,4) when the arc beginning at the vernal point extends over more than one sign.

One now uses the same diagram as representation of the plane of the day-circle by making OE = OF = DB = r. We then determine on the day-circle a point F such that the chord EF = $2e$. If we call the resulting angle

$$\mathrm{EOF} = 2\omega$$

one has

$$\sin\omega = e/r$$

thus with (1)

$$\mathrm{Sin}\,\omega = \frac{R}{r}\,\mathrm{Sin}\,\delta\,\tan\varphi \tag{2}$$

which is, according to IV,26 (2), the defining relation for the ascensional difference ω which belongs to δ.

The present method is, of course, independent of the units in which the radius R of the diagram is drawn. Why the text requires to make OA = 90 digits is not clear since $R = 90$ is nowhere else a norm for trigonometric functions. Only if our first interpretation of XIV,7 is correct the number 90 could perhaps represent the number of degrees from the pole to the equator.

XIV,5–6. If t is a time interval (since sunrise or before sunset), measured in nāḍīs, then $6t$ is the equivalent angle in degrees. If furthermore s_0 is the length of the shadow cast by a vertical gnomon of length g at noon, s at any time t, the text seems to tell us how to find the increment of s over s_0. Apparently this should be found from

$$s = s_0 + R - \mathrm{Sin}\,t. \tag{1}$$

Unfortunately such a relation cannot be correct since it gives for $t = 0$ not $s = \infty$ but $s = s_0 + R$ (though for $t = 90°$ one obtains $s = s_0$ as expected).

Furthermore any such formula must involve g, the geographical latitude φ, and the solar declination on the day in question. The only possibility of avoiding these data would imply that (1) concerns only the equinoxes and that s_0, the equinoctial noon shadow, is considered to be a given parameter. But even so s would have to be found from $s = s_0/\cos\alpha$ with α found from $\tan\alpha = \cot r/\sin\varphi$.

XIV,7. A possible interpretation of this verse is illustrated by Fig. 49. Let AN represent a quadrant of the equator with OA = 90 (cf. XIV,1–4). On this quadrant make AP = 30°, AQ = 60°. Parallel to NO are drawn the declinations NB = ε, QQ′ = $\delta(60°) \approx 20;30°$, PP′ = $\delta(30°) \approx 11;40°$, measured in the same units as the radius NO—and, if desired, analogously for any other arc from A toward N. The resulting curve AP′Q′B is then considered as representing the ecliptic. One now draws the radii OP′P″, OQ′Q″ which intersect the given quadrant at points P″, Q″ which define arcs AP″ = $\alpha(30°)$, AQ″ = $\alpha(60°)$ which are supposedly the right ascensions for $\lambda = 30°$ and $\lambda = 60°$ respectively.

That this procedure is not correct (except for A and N) is obvious. This can be seen also by direct measurement which gives AP″ $\approx 23°$ (instead of $\approx 27{:}50°$) and AQ″ $\approx 52°$(instead of $\approx 57{;}55°$). Only the general trend of the right ascensions as function of λ is more or less preserved. Cf. also XIV,10–11.

The advantage of this interpretation of the text lies in the fact that it relates the problem at hand to the general type of numerical and graphical methods which are presented in this chapter. Otherwise an alternative translation could be proposed which implies no more than a simple description of the concept "right ascension":

"The ecliptic (lies) on a line (running through) the degrees of declination north and south of the equator; the degrees of the (corresponding) arcs of that (i.e., of the

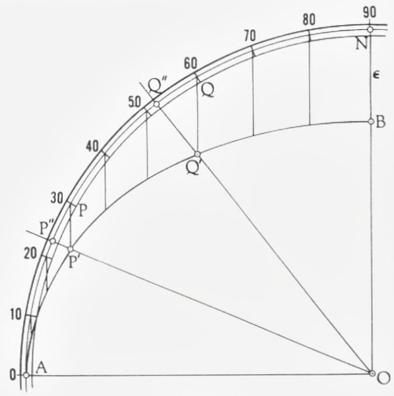

Fig. 49

equator) multiplied by 10 are, in order, the vināḍikās if rising (the right ascensions) of the zodiacal signs".

XIV,8–10. Let G be the foot of the gnomon of length g (Fig. 50), the point C of the north-south line the endpoint of the equinoctial noon shadow. A point F is obtained by turning the gnomon down into the east-west line, hence making FG $= g$. Then FC intersects the circle at an angle $\bar{\varphi}$ from CG such that $\bar{\varphi}$ is the colatitude.

The construction in XIV,9–10 (cf. Fig. 51) is only a trivial variant of the same idea.

XIV,10–11. Based on the graphical method assumed in Fig. 49 as interpretation of XIV,7 the longitude λ which belongs to a given declination δ could be found by fitting δ (parallel to NBO) between the two curves (cf. Fig. 52). The principle of such a procedure is, of course, incorrect, e.g., because the quadrant AN is now used as representing longitudes though originally serving as the equator.

XIV,12–13. The elongation $\Delta\lambda$ between Sun and Moon is found by direct observation. Since the velocity of the Moon relative to the Sun is about $12°$ per tithi the quotient $\Delta\lambda/12$ is an estimate of the number of elapsed tithis since conjunction.

XIV,14–16. The following construction supposedly provides a method for finding the path of the shadow end of a vertical gnomon and the position of the meridian line from three arbitrarily selected positions of the shadow (cf. Fig. 53). If G is the foot of the gnomon, A, B, C the endpoints of three shadows s_1, s_2, s_3 then M is constructed as midpoint of the circle which circumscribes the triangle ABC; this circle is assumed to be the path of the shadow, i.e. a hyperbola is approximated by a circular arc as may be reasonably adequate for the section from A to C.

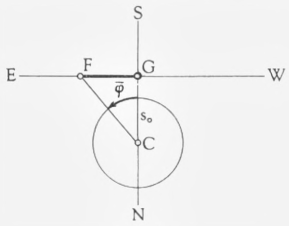

Fig. 50.

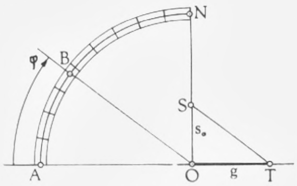

Fig. 51.

It is obviously wrong, however, that MG defines the meridian line since the same points A, B, C, and M can be the result of totally different positions of G. Or formulated differently: three points define a circle but not a hyperbola.

XIV,17–18. Definition of concepts which have been used many times before.

XIV,19–20. Construction of a hemispherical sun dial (cf. Fig. 54), the plane of its rim being made to coincide with the plane determined by the east-west line and the direction to the north pole. The gnomon in the center will point in the direction of midheaven, its tip being at the center of the sphere. The endpoint of the shadow then describes the part above the horizon of the respective day-circle. From this can di-

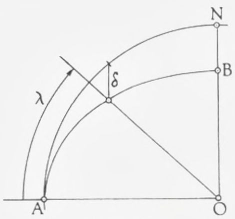

Fig. 52.

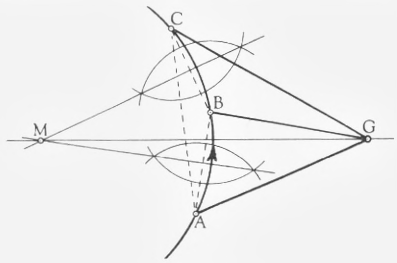

Fig. 53.

rectly be known the nāḍīs that have elapsed since sunrise. However, one cannot find the ascendent point of the ecliptic by adding an arc on the day-circle to the longitude of the Sun.

XIV,21–22. Fig. 55 illustrates the principle of the instrument in question. Using for the "hand" (hasta) and the "finger" a norm as found in the Āryabhaṭīya (Daśagītikā

6) we get a ratio 1:48 for the width of the ring to its diameter and width of the ring of about 1/3 of an inch. To keep such a ring accurately circular and in the same plane will not be easy. A practical execution of the instrument was probably never attempted.

XIV,23–26. The text seems to suggest a solid sphere with the principal circles drawn on its surface. Equidistant perforations made on the ecliptic should then be used to single out two diametrically opposite holes which define the position of the Sun in longitude.

The practical difficulties in constructing such a sphere would be extremely great and an armillary sphere, consisting of a system of rings, would serve the purpose

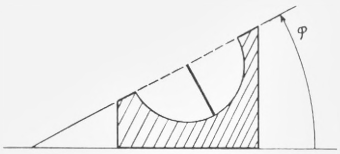

Fig. 54.

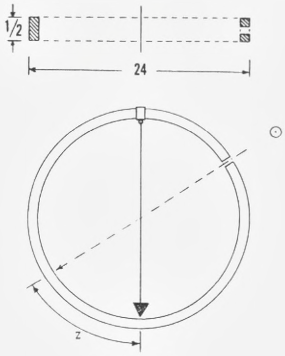

Fig. 55.

much better. The wording of the text, however, seems not to allow such an interpretation.

XIV,27–28. Strings are used for drawing circles, water for leveling surfaces, sand either for constructing diagrams on the ground or for computing on the sand-board.

The references to the forms of tortoises and men look more like tantra than gaṇita.

XIV,29–30. The only accurate time signals available in ancient astronomy for the determination of relative geographical longitudes were lunar eclipses.[1] The present verses speak of full-moons in general which would, in practice, deprive the method of all its value since the accurate moment of opposition cannot be established by observation.[2] Nevertheless the underlying idea is in principle correct, being based on a reversal of the rule for the application of the correction for geographical longitude-differences, e.g., in IX,10.

The longitudes of Sun and Moon at opposition, i.e., λ_s and $\lambda_m = \lambda_s + 180°$ may be assumed to be known by computation. Hence one can in principle establish by

[1]) This method can be made independent from man-made clocks by relating the characteristic phases of the eclipse to transit observations of stars.

[2]) An error of only 10 minutes in longitude (considered permissible even at the height of Greek astronomy) would result in an error of 5° in geographical longitude and parallax alone could increase the error to five times this amount.

observation the moment at which the moon reaches this longitude λ_m. At that moment one also should observe which point of the ecliptic is rising, giving us, at least ideally, a longitude λ_H and hence an ecliptic arc $\lambda_H - \lambda_m$ which is equal to the arc $\lambda_s - (\lambda_H + 180°)$ The oblique ascension of the arc $\lambda_H - \lambda_m$ is therefore equal to the time t' (expressible in ghaṭikās) since local sunset.

The simplest method of proceeding would be to apply the equation of daylight to t' and thus to find the time t of the observation after 6 p.m. (in equinoctial hours). On the other hand, one can find by the standard computational methods of the karaṇa the time t_0 of the opposition with respect to 6 p.m. Laṅkā. Hence the time difference $\Delta t = t_0 - t$ between the given locality and the prime meridian would be known, which solves our problem.

The text is by no means clear but seems to ask for a more intricate procedure. Although mentioned in the text at the wrong place we may assume the transformation of t' into t carried out correctly, i.e., we consider to be known the time t in equinoctial hours after 6 p.m. local time at which the opposition took place. One now computes for the same number t of hours after 6 p.m. Laṅkā time the longitude λ_0 of the Moon. Then $\Delta\lambda = \lambda_m - \lambda_0$ represents the lunar motion during the time interval $\Delta\lambda$ between the local meridian and the prime meridian through Laṅkā. Hence Δt can be found from $\Delta\lambda$ by dividing the latter by the lunar velocity-reminiscent of the conversion of a longitudinal difference into tithis.

This circuitous way is not only needlessly complicated but increases considerably the number of inaccurately known elements. The whole method probably had never been tried in practice.

XIV,31–32. Both the inpouring and the outpouring types of waterclocks are here described. For other references in early Sanskrit literature see Isis 54 (1963) p. 232 to which add Sphujidhvaja, Yavanajātaka 79,27.

The text states that there are 180 breaths (śvāsa) in a nāḍī. Each śvāsa consists of two prāṇas—an ingoing and an outgoing. The prāṇa, in fact, is traditionally a sixth of a vināḍī (i.e. $^1/_{360}$ of a day) or the time necessary to recite 10 long syllables. XIV,32 contains 60 long syllables, and should therefore require 1 vināḍī to be recited.

Table 10.

Yogatārā	Longitude		Latitude		No.
	Varāhamihira	Paitāmaha	Varāhamihira	Paitāmaha	
Kṛttikā	32;40°	37;28°	+3;9°	+5°	1
Rohiṇī	48;30	49;28	−5;51	−5	2
Punarvasu	88	93	+7;12 and −7;12	+6	3
Puṣya	97;20	106	+4;3	0	4
Āśleṣā	107;40	108	+0;54 and −0;54	−7	5
Maghā	126	129	–	0	6
Citrā	181;50	183	−2;42	−2	7

Table 11.

Regulus	
A.D.	λ
0	122; 9°
100	123;32
200	124;54
300	126;17
400	127;40
500	129; 3
600	130;26

Table 12.

No.	Longitude		Latitude	
	Var.	Pait.	Var.	Pait.
1	−93;20°	−91;32	+3; 9°	+5°
2	−77;30	−78;32	−5;51	−5
3	−38	−36	±7;12	+6
4	−28;40	−23	+4; 3	0
5	−18;20	−21	±0;54	−7
6	0	0	–	0
7	+55;50	+54	−2;42	−2

XIV,33–37. These verses are to be used in predicting the conjunctions of the Moon with certain fixed stars whose longitudes are given as degrees within the nakṣatras for which they serve as reference stars ("yogatārās"); the latitudes are given in "hands" (hastas), of which each should be approximately 0;54°.

We can easily compare (cf. Table 10) the figures in our text with those of the Paitāmahasiddhānta of the Viṣṇudharmottarapurāna (III,30) from which all later Indian star catalogues seem to be derived—with various modifications, of course. The discrepancies in latitude seem to indicate that the yogatārās of Kṛttikā, Puṣya, and Āśleṣā according to Varāhamihira are different from those according to Paitāmaha.

The safest identification seems Maghā and Regulus (α Leo) since the longitudes of Regulus between A.D. 0 and 600 agree with such an identification (cf. Table 11). The latitude of Regulus during this period is practically constant +0;24° in fair agreement with the Indian norm.

For the further investigation it is advantageous to operate with relative longitudes, e.g., with respect to Maghā, since one eliminates in this way all chronological questions. Thus Table 12 can replace Table 10. Similarly Table 13 gives the coordinates of six stars which may be identified with some degree of plausibility with the yogatārās Nos. 1 to 3 and 5 to 7 of Table 12. For No. 4 one would like to suggest some star in Cancer but no convincing identification presents itself.

Table 13.

	Magn.	Long.	Lat.	No.
η Tau (Hyades)	5	−89;57°	+3;52°	1
α Tau (Aldebaran)	1	−80;11	−5;35	2
β Gem (Pollux)	2	−36;27	+6;33	3
ε Can (Crib)	neb.	−22;32	+1; 0	5
α Leo (Regulus)	1	0	+0;24	6
α Vir (Spica)	1	+53;55	−1;56	7

XIV,38. If the Moon at latitude β_m comes in conjunction with a star of latitude β, i.e., if the Moon and the star are of same longitude, then the distance b of the northern rim of the Moon from the star (cf. Fig. 56) is given by

$$b = (\beta - \beta_m) - 0;17^\circ \tag{1}$$

since (VI,3) we have for the apparent diameter of the Moon

$$d_m = 0;34^\circ. \tag{2}$$

If one measures, however, the distance b in "digits" such that

$$d_m = 15 \text{ digits} \tag{3}$$

then one has to replace (1) by

$$b = \tfrac{15}{34}((\beta - \beta_m)' - 17') \text{ digits} \tag{4}$$

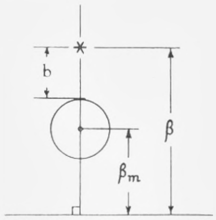

Fig. 56.

measuring β and β_m in minutes of arc. According to XIV,33 the distance $\beta - \beta_m$ has been found by observation. Of course it would be more reasonable to determine β_m from the directly observable distances β and b.

The origin of the norm (3) is unknown but in V,4 we have a similar division of the lunar diameter in 15 "parts".

XIV,39–41. It is the purpose of the text to compute for a given geographical latitude φ the longitude λ_s of the Sun at the heliacal rising of the star Canopus (which has a longitude near, or equal to Cancer 0° i.e. 90°). The text relates the rising time ϱ of the arc $\lambda_s - 90^\circ$ to an expression reckoned in degrees and then multiplied by 10 in order to obtain time units (vināḍīs). We can avoid this factor by counting also oblique ascensions in degrees. Then the text seems to say

$$\tfrac{1}{2}(s_0 \cdot 0;25^\circ + 21^\circ \cdot s_0) = \varrho(\lambda_s - 90^\circ) \tag{1}$$

where s_0 is the equinoctial noon shadow.

$$s_0 = 12 \tan\varphi \tag{2}$$

and thus for (1)

$$2,8;30 \tan\varphi = \varrho(\lambda_s - 90^\circ). \tag{3}$$

This relation cannot be strictly correct since we know that the Sun must be somewhere in the last third of Leo[1]) which makes $\lambda_s - 90°$ about 50° or 60°, hence the oblique ascension greater than 50 and increasing with φ. The left-hand side, however, is zero at the equator and increases toward 120 as φ moves toward 45°. Hence (3) can be only approximately correct within certain limits of φ. It is plausible to investigate the situation for $\varphi = 24°$, which is the latitude of Ujjayinī, hence for $\tan\varphi \approx 0;27$. This gives

$$\varrho(\lambda_s - 90°) \approx 58$$

which is satisfied at $\varphi \approx 24°$ by $\lambda_s \approx$ Leo 21. Hence (1) would be applicable for an area which contains Ujjayinī.

Chapter XV

XV,1–4. The Sun is "eclipsed" for any observer located within the shadow of the Moon. In this sense there is always a solar eclipse somewhere in the universe; he who correctly knows the relative position of the two luminaries can predict where

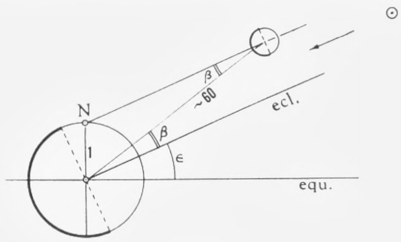

Fig. 57.

and when a solar eclipse will be visible. The Pitṛs, who dwell on the side of the Moon opposite the earth, cannot see the Sun for half a synodic month; the middle of this type of "solar eclipse" occurs at full moon.

XV,5–6. On the north pole solar eclipses are considered impossible. The argument seems to be that the ecliptic is not far enough from the equator to cause the Moon's shadow to reach the north pole. This would be, however, the case only if the Moon would be restricted to a position in the ecliptic. But it is easy to see (cf. Fig. 57) that for a conjunction near the summer solstice a lunar latitude of about 1° suffices to produce a solar eclipse on the north pole.

XV,7–9. Dependence of a solar eclipse on local time, whereas XV,9 refers to the influence of longitudinal parallax, here estimated as reaching as much as 2 kṣaṇas = 4 nāḍīs. This agrees with the estimate for the horizontal parallax in VII,1 and VIII,9.

XV,10. The reference is to Bṛhatsaṃhitā 5, 8–11.

[1]) Bṛhatsaṃhitā XII,14 gives Leo 23° as solar longitude when Canopus rises.

XV,11–14. If one calls a "nychthemeron" the succession of one period of light and one of darkness, then 60 nāḍīs is a nychthemeron for men, a synodic month for the Pitṛs on the Moon, and one year for the Gods on the north pole.

XV,15. The maximum altitude at which the Gods on the north pole can see the Sun is $\varepsilon = 24°$. In contrast, the same altitude is attained in 2 kṣaṇas = $^1/_{15}$ day when the Sun crosses a horizon perpendicularly, as is possible for geographical latitudes $|\varphi| \leqq \varepsilon$, thus, e.g., anywhere between Ujjayinī and Lañkā.

XV,16–21. This discussion is clear enough; it will suffice to indicate here some parallel references to statements about the epochs of the various authorities listed by Varāhamihira.

XV,18. The epoch here ascribed to Lāṭācārya is found again in I,8 as that of the Romakasiddhānta; the epoch-date 22 March 505 A.D., then, must be Lāṭa's. But note that in VIII,5 sunset at Avantī is ascribed to the Romakasiddhānta.

XV,19. Siṃhācārya is not otherwise known, but his epoch is the standard epoch of most Indian astronomers—e.g., for both the āryapakṣa and the brāhmapakṣa.

The guru of the Yavanas gives as epoch 10 muhūrtas i.e., 8 hours past sunset, hence about 2 hours $\approx 30°$ to the west of the prime meridian Lañkā-Ujjayinī. We have seen in III,13 that Yavanapura (Alexandria) is assumed to lie 44° to the west of Ujjayinī, as is very nearly correct. Baghdad-Babylon, however, lies 31;25° west of Ujjayinī; so the epoch of the guru of the Yavanas is midnight at Babylon as it was, e.g., in the Sasanian Zīj ash-Shāh (according to al-Bīrūnī, cited by E. S. Kennedy, JAOS 78, 1958, p. 260–261).

XV,20. Āryabhaṭa's first epoch is that of the ārdharātrika system, for which see Part I p. 14; his second is that of the Āryabhaṭīya, Daśagītikā 2 and Kālakriyā 16.

XV,22–23. In Purāṇic cosmology the inner continent, Jambūdvīpa, of which Mount Meru is the center, has at its four cardinal points the four territories listed here: Bhāratavarṣa (i.e., India) to the south, the Bhadrāśvas to the east, the Kurus to the north and the Ketumālas to the west. Similarly the astronomers (e.g. Āryabhaṭīya, Gola 13) posit on the equator four cities distant 90° from each other; starting from Lañkā and proceeding westward they are Lañkā, Romakaviṣaya, Siddhapura, and Yamakoṭi.[1])

XV,24. This verse states that the various series of chronological units begin at the beginning of the yuga, at which time all of the planets were at Aries 0°. There is some ambiguity in the text as it is stated that both the series of days and that of nights begin at this time. The apparent contradiction disappears if we assume a sunset epoch and interpret the word "day" (dina) in the sense of "nychthemeron". But Varāhamihira may also have been expressing himself in a very loose manner.

[1]) Cf. D. Pingree, The Thousands of Abū Maʿshar, p. 45.

The ayanas are usually the semicircles of the solar orbit from solstice to solstice but in the present context a beginning at the equinoxes is required. The ṛtus define a kind of seasonal division of the year. The "motion of the constellations" counts the number of sidereal days in a yuga.

XV,25. Romakaviṣaya is per definition 90° west of Lañkā, and Yavanapura is by III,13 44° west of Lañkā. The variation referred to in the second half of the verse is a direct effect of the variation in the length of daylight; such a variation does not exist, however, at Lañkā itself but at any other locality on the prime meridian through Lañkā.

XV,26–29. Criticism of the ordinary concept "Lord of the day" because of its dependence on geographical location.

Chapter XVI

XVI,1–9. In order to find from the ahargaṇa a for the outer planets their mean (sidereal) longitude $\bar{\lambda}$, for Venus and Mercury the śīghra (i.e. the sum of the mean sidereal longitude of the Sun plus the planet's mean anomaly), rules of the following form are given

$$\bar{\lambda} = 360 \frac{a}{p_0} + \frac{a}{p_0} \delta + c. \qquad (1)$$

Here p_0 is the approximate duration (in days) of one sidereal rotation, δ a correction due to the inaccuracy of p_0, and c (in degrees) the epoch constant (kṣepa).

The given data for $1/p_0$ and the resulting values for p_0 are shown in Table 14. Note that the values of p_0 are the exact equivalents of the ratios given for $1/p_0$, no roundings being involved. The identical values for $1/p_0$ for Mars, Venus, and Mer-

Table 14.

	$1/p_0$	p_0
Saturn	$\frac{1000}{10766066} = \frac{16,40}{49,50,34,26} = \frac{1,0,0,0}{2,59,26,3,57,36}$	$2,59,26;3,57,36^d$
Jupiter	$\frac{100}{433232} = \frac{1,40}{2,0,20,32} = \frac{1,0,0}{1,12,12,19,12}$	$1,12,12;19,12^d$
Mars	$\frac{1}{687} = \frac{1}{11,27}$	$11,27^d$
Venus śīghra	$\frac{10}{2247} = \frac{10}{37,27} = \frac{1,0}{3,44,42}$	$3,44;42^d$
Mercury śīghra	$\frac{100}{8797} = \frac{1,40}{2,26,37} = \frac{1,0,0}{1,27,58,12}$	$1,27;58,12^d$

Table 15.

	N		Np_0	$\Delta = Np_0 - A$
Saturn....	146 564 =	40,42,44	2,1,45,10,28,17;13,26,24^d	− 1,42;46,33,26^d
Jupiter ...	364 220 =	1,41,10,20	2,1,45,10,26,30;24	− 3,29;36
Mars	2 296 824 =	10,38, 0,24	2,1,45,10,34,48	+ 4,48
Venus	7 022 388 =	32,30,39,48	2,1,45,14, 3, 3;36	+3,33,3;36
śīghra Mercury ..	17 937 000 =	1,23, 2,30, 0	2,1,45,10,31,30	+ 1,30
			2,1,45,10,30,0 = A	

cury and equivalent expressions for Saturn and Jupiter are found in Khaṇḍakhādyaka II,1–5.

In order to determine the correction δ one has to introduce the numbers p of days which represent the accurate lengths of the sidereal periods of the planets. Traditionally these numbers p are defined by saying that exactly N sidereal revolutions of the planet take place during A days, i.e.,

$$p = \frac{A}{N}. \tag{2}$$

For the number A can be taken the number of days in a Mahāyuga, i.e., in 20,0,0,0 sidereal years. In the ārdharātrika system it is assumed (cf. IX,1) that

$$A = 1\ 577\ 917\ 800^d = 2,1,45,10,30,0^d \tag{3}$$

which is the exact equivalent of the statement that

$$1 \text{ sidereal year} = 365;15,31,30^d. \tag{3a}$$

In the following we shall use the parameter (3) and the values N listed in Table 15.

It follows from the definition of p as exact sidereal period that one should have

$$\bar{\lambda} - c = 360\frac{a}{p}$$

whereas (1) gives

$$\bar{\lambda} - c = 360\frac{a}{p_0} + \frac{a}{p_0}\cdot\delta.$$

Consequently

$$\frac{a}{p_0}(360+\delta) = \frac{a}{p}360 \text{ and thus } \delta = 360\left(\frac{p_0}{p} - 1\right).$$

Finally, using (2), we find for δ, measured in degrees

TABLE 16.

	$6,0 \cdot \Delta / A$	Text: δ
Saturn	$-0;\ 0,\ 0,\ 5,\ 3,\ldots^\circ$	$-0;\ 0,\ 0,\ 5^\circ$
Jupiter	$-0;\ 0,\ 0,10,19,\ldots$	$-0;\ 0,\ 0,\ 10$
Mars	$+0;\ 0,\ 0,14,11,\ldots$	$+0;\ 0,\ 0,14$
Venus	$+0;\ 0,10,29,58,\ldots$	$+0;\ 0,10,30$
śīghra		
Mercury ...	$+0;\ 0,\ 0,\ 4,26,\ 6,\ldots$	$+0;\ 0,\ 0,\ 4,\ 30$

$$\delta = 360 \frac{Np_0 - A}{A} \tag{4}$$

in excellent agreement with the values found in the text (cf. Tables 15 and 16).

The problem of determining the accurate moment of the epoch is naturally related to the explanation of the values given in the text for the epoch constants, the kṣepas. We shall show in the following that the epoch positions for the Sun and the Moon are referred to noon (Ujjayinī) of March 20 A.D. 505 whereas the positions for the planets are based on midnight March 20/21 of that year. One can consider this inconsistency as evidence for an earlier version of the Sūryasiddhānta (cf. above Pt. 1 p. 13 f.)

For the date of the epoch we can conclude from the rule given in I,8 for the computation of the ahargaṇa that 427 years in the Śaka era were completed at the epoch. Adding 427 to the number 3179 af years conventionally assumed as the date for the beginning of the Śaka era with respect to the Kaliyuga[1]) we obtain for our epoch a distance of exactly 3606 (= 1,0,6) years from the beginning of the Kaliyuga.

We know furthermore (cf. (3a)) that

$$1 \text{ sidereal year} = 6,5;15,31,30^{d}.$$

Consequently the 1,0,6 years elapsed since the beginning of the Kaliyuga contain

$$6,5;15,31,30 \cdot 1,0,6 = 6,5,52,3;3,9^{d} \approx 1\,317\,123^{d}. \tag{5}$$

If we add these days to the beginning of the Kaliyuga, i.e. to midnight of February 17/18 −3101 we obtain (ignoring the fraction $0;3,9^{d}$) midnight March 20/21 A.D. 505.

Computing for this moment and with the parameters of the ārdharātrika system the planetary (mean) positions one finds exactly, or almost exactly, the epoch constants given in the present chapter (cf. Table 17). For Sun and Moon, however, one finds discrepancies of about one half day's motion too short; this shows that for these bodies noon of March 20 had been used as epoch, thus confirming the statement in IX,1 that noon in Avantī is the point of reference for the solar longitudes. In the following we shall first deal with the epoch constants for Sun and Moon as given in

[1]) Brāhmasphuṭasiddhānta I,26 or Laghubhāskarīya I,4.

Table 17.

	N	0;18,1,48 · N = c_1	$\bar{v}$	− 0;3,9 · $\bar{v}$ = c_2	$c_1 + c_2 = c'$	c	XVI	c − c′
Saturn . .	40,42,44	122;28,55,12°	0; 2, 0,23°/d	−0; 0, 6,19°	122;28,48,53°	122;28,49°	5	0
Jupiter .	1,41,10,20	8; 6,36, 0	0; 4,59, 9	−0; 0,15,42	8; 6,20,18	8; 6,20	6	0
Mars . . .	10,38, 0,24	75;36,43,12	0;31,26,27	−0; 1,39, 2	75;35, 4,10	75;35	6	−0;0,4°
Venus . .	32,30,39,48	267;35,38,24	1;36, 7,44	−0; 5, 2,48	267;30,35,36	267;30,39	9	+0;0,3
Mercury	1,23, 2,30, 0	148;30, 0, 0	4; 5,32,17	−0;12,53,27	148;17, 6,33	148;17	9	−0;0,7

IX,4 and 5 (cf. Table 18 and 19) before turning to the planetary kṣepas from XVI,5, 6, and 9 (Table 17).

Let N again represent the number of sidereal rotations of a celestial body (or apogees, or nodes) in 20,0,0,0 sidereal years, thus $N/20,0,0,0$ the number of revolutions per year and therefore

$$1,0,6 \cdot \frac{N}{20,0,0,0} \cdot 6,0^\circ = 0;18,1,48 \cdot N \tag{6}$$

the mean motion during 1,0,6 sidereal years since the beginning of the Kaliyuga. At the beginning of the Mahāyuga, i.e. 15,0,0,0 years before the beginning of the Kaliyuga, all mean longitudes are assumed to be zero. The number of sidereal revolutions during 15,0,0,0 years is given by

$$15,0,0,0 \cdot \frac{N}{20,0,0,0} = 0;45 \cdot N. \tag{7}$$

Hence, whenever the last digit of N, multiplied by 0;45, produces a number ending in zero—that is to say, whenever the last digit of N is divisible by 4—then the number of rotations during 15,0,0,0 years is an integer and therefore the corresponding longitude will again be zero, also at the beginning of the Kaliyuga.

The values of N for the rotations of Sun and Moon in a Mahāyuga are[1])

Sun, mean longitude:	20,0,0,0
Moon, mean longitude:	4,27,22,35,36
Moon, apogee:	2,15,36,59
Moon, nodes (retrograde):	1,4,30,26.

The first two numbers are divisible by 4; thus the mean longitude of Sun and Moon is zero at the beginning of the Kaliyuga. But 59·0;45 = 44;15 and 26·0;45 = 19;30. Consequently the lunar apogee had a longitude of 90° at the beginning of the Kaliyuga while the longitude of the ascending node was ±180°.

[1]) Cf. note 1 to I,14, Table 1.

TABLE 18.

	$c_0 + 0;18,1,48 \cdot N = c_1$
Sun, Mean Long.	0° + 0° = 0°
Moon, Mean Long.	0 + 357;28, 4,48 = −2;31,55,12°
Moon, Apogee	90 + 189;48,34,12 = 279;48,34,12
Moon, Asc. Node	180 − 303;54,46,48 = −123;54,46,48

Having thus found the initial longitudes c_0 for the beginning of the Kaliyuga we can now go 1,0,6 years forward to the epoch in A.D. 505 by adding, according to (6), the amount of $0;18,1,48 \cdot N$. The resulting longitudes c_1 are shown in Table 18. We must now observe that (5) tells us that 1,0,6 sidereal years, beginning at midnight of −3101 Febr. 17/18 lead $0;3,9^d$ beyond midnight of A.D. 505 March 20/21. Thus noon of March 20, the epoch for Sun and Moon, precedes by $0;33,9^d$ the endpoint of 1,0,6 years.

Let $\bar{v}$ be the daily mean motions as determined from the parameters of the ārdharātrika system in IX,1–5. Then

$$c_2 = -0;33,9 \cdot \bar{v} \tag{8}$$

furnishes the motion away from c_1 found before. Thus

$$c' = c_1 + c_2 \tag{9}$$

are the longitudes to be expected for the epoch, i.e., for noon of March 20 A.D. 505. Table 19 shows that the agreement with the kṣepas given in IX,4 and 5 is excellent.

A similar consideration leads us to the kṣepas for the planets in XVI,5–9. All numbers N are divisible by 4, thus all longitudes are zero at the beginning of the Kaliyuga and (6) gives directly c_1 (cf. Table 17). Since midnight epoch is used for the planets the excess of c_1 is only $0;3,9 \cdot \bar{v}$ where $\bar{v}$ is again the daily mean motion based on ārdharātrika parameters. Thus the expected epoch-longitudes are $c' = c_1 - 0;3,9 \cdot \bar{v}$, again in excellent agreement with the kṣepas in the text.

Table 20 gives in column I the modern data for the planetary positions in A.D. 505 March 20 (7 p.m. Babylon).[1] Column II is computed with Theon's "Handy

TABLE 19.

	IX	$\bar{v}$	$-0;33,9 \cdot \bar{v} = c_2$	$c_1 + c_2 = c'$	c	IX	$c - c'$
Sun, Mean Long.	1 (3)	0;59,8,10°/d	−0;32,40,22°	−0;32,40,22°	−0;32,40°	1 (5)	0
Moon, Mean Lng.	2 (3)	13;10,34,52	−7;16,47,46	−9;48,42,58	−9;48,44	2 (4)	−0;0,1°
Moon, Apogee	3 (3)	0;6,40,59	−0;3,41,33	279;44,52,39	279;44,53	3 (4)	0
Moon, Asc. Node	5 (3)	−0;3,10,44	+0;1,45,23	−123;53,1,25	−123,53,3	5 (4)	−0;0,2

[1]) From Tuckerman, Tables. The time difference to midnight Ujjayinī is only 3^h.

TABLE 20.

Planet		I	II	c	c – II
Saturn	λ	123.19°	120;13°		
	$\bar{\lambda}$		125;19	122;28,49°	– 2;50°
Jupiter	λ	9.41	7;45		
	$\bar{\lambda}$		9;41	8; 6,20	– 1;35
Mars	λ	53.78	47;31		
	$\bar{\lambda}$		80;34	75;35	– 4;59
Venus	λ	324.28	320;53		
	$\bar{\lambda}$		359;17		
	$\bar{\alpha}$		265; 2	267;30,39	+ 2;29
Mercury	λ	6.89	13; 1		
	$\bar{\lambda}$		359;17		
	$\bar{\alpha}$		149;12	148;17	– 0;55

Tables" [1]) (for midnight Ujjayinī, March 20/21). The true longitudes in column II differ so little from the modern values that we may consider the mean longitudes and anomalies as fair estimates for these parameters (which have no simple parallel in modern tables). The agreement with the kṣepas *c* is close enough to show their correctness for the date of the epoch.

For the Moon all relevant parameters can conveniently be computed with P. V. Neugebauer's tables (cf. Table 21, column II) and, for the sake of comparison, also from the "Handy Tables" (Table 21, column I). The longitude λ_A of the apogee of the lunar orbit can be found, according to Fig. 58, from

$$\lambda_A = \bar{\lambda} - \bar{\alpha}. \tag{10}$$

The agreement of the kṣepas is very good indeed.

TABLE 21.

Moon	505 March 20 noon Ujj.		c	c – II
	I	II		
$\bar{\lambda}$	– 9;24°	– 9.84°	– 9;48,44	+ 0;1,40°
λ	345;50	345.13		
$\bar{\alpha}$	74; 5	70.6	70;26,23	– 0;9,37
λ_A	276;31	279. 6	279;44,53	+ 0;9
$-\lambda_n$	126;57	123.8	123;53,3	+ 0;5
β	+ 4;36	+ 4;45		

[1]) Using (after correcting many errors) the edition by Halma "Commentaire de Théon d'Alexandrie...; Tables manuelles des mouvemens des astres" Paris 1822, 1823.

XVI,10–11. The yearly corrections (bīja) required according to these verses are

Saturn	mean long.	+0;0,6,30°	Venus	śīghra	−0;0,45°
Jupiter	mean long.	−0;0,10	Mercury	śīghra	+0;2
Mars	mean long.	+0;0,17			

For Jupiter an additional correction of −0;23,20° (at epoch?) is to be applied.

If one computes the number of revolutions which accumulate by these corrections during 20,0,0,0 years and then adds the results (algebraically) to the numbers N known from the ārdharātrika system one obtains numbers (involving fractions for the outer planets) which are recorded nowhere else. We have no explanation to offer.

XVI,12–14. The underlying model of planetary motion assumed by the Sūrya-siddhānta, as by other early Indian texts, is a deferent concentric with the center O

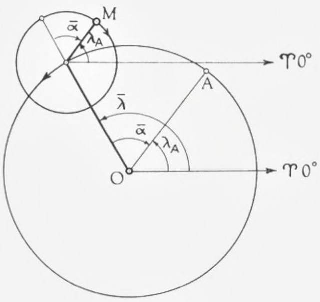

Fig. 58.

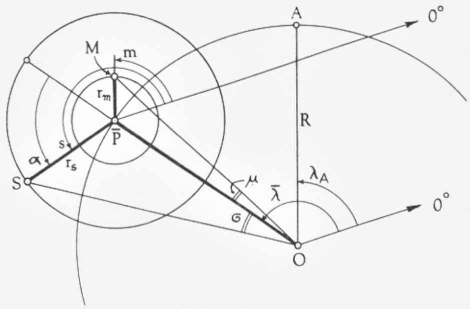

Fig. 59.

of the earth (cf. Fig. 59), carrying the mean planet $\overline{\mathrm{P}}$. The latter is the center of two epicycles, the "manda-epicycle" and the "śīghra-epicycle". On the former is situated the "mandocca" M such that the radius $r_{\mathrm{m}} = \overline{\mathrm{P}}\mathrm{M}$ has a fixed sidereal direction, parallel to the direction from O to the apogee A; on the latter epicycle is moving the "śīghrocca" S such that, for an outer planet, $\overline{\mathrm{P}}\mathrm{S} = r_{\mathrm{s}}$ is always parallel to the direction from O to the Sun (which, then, the text calls śīghra, i.e., "conjunction") whereas for an inner planet $\overline{\mathrm{P}}$ coincides with the mean Sun while $\overline{\mathrm{P}}\mathrm{S}$ makes with the direction $\mathrm{O}\overline{\mathrm{P}}$ the angle α which represents the anomaly of the planet.

Obviously the displacement caused by the manda-epicycle is the cinematic equivalent of an eccentric deferent in the Greek planetary theory (with $\overline{\mathrm{P}}\mathrm{M}$ as eccentricity) whereas the śīghra epicycle plays essentially the same role as the epicycle which carries the planet.

As independent variables serve in the Indian arrangement the angles m (manda) and s (śīghra) shown in Fig. 59,

$$m = \lambda_{\mathrm{A}} \tag{1}$$

being the (sidereal) longitude of the apogee A, and

$$s = \bar{\lambda} + \alpha, \tag{2}$$

the sum of the mean longitude of the planet and its anomaly. Both m and s produce corrections, μ and σ respectively, which, combined and modified in a fashion to be described presently, lead from the mean longitude $\bar{\lambda}$ of the planet to its true longitude λ.

The basic parameters for both inequalities are expressed in the text as circumferences of the epicycles, c_m and c_s respectively, measured in units of which the circumference c of the deferent of radius R contains 360.[1]) Since

$$\frac{r_m}{R} = \frac{c_m}{360} \qquad \frac{r_s}{R} = \frac{c_s}{360} \tag{3}$$

and since we have in the Pañcasiddhāntikā the norm $R = 120$, it follows from (3) that

$$r_m = \frac{c_m}{3} \qquad r_s = \frac{c_s}{3}; \tag{4}$$

for the norm $R = 60$ adopted in Greek astronomy, however, one has

$$e = \frac{c_m}{6} \qquad r = \frac{c_s}{6}. \tag{5}$$

Table 22 shows the specific values (following the ārdharātrika system) for the individual planets.[2]) All longitudes λ_A of the apogees are sidereally fixed. It should be noted that the manda-parameters of Venus are the same as the solar parameters, taken from IX,7–8 (above p. 69 f.). This means, expressed in modern terms, that the solar orbit can serve as deferent for Venus.

TABLE 22.

	manda				śīghra			asc. node
	c_m	r_m	e	λ_A	c_s	r_s	r	λ_n
Saturn	60	20; 0	10; 0	240°	40	13;20	6;40	100°
Jupiter	32	10;40	5;20	160	72	24	12	80
Mars	70	23;20	11;40	110	234	78	39	40
Venus	14	4;40	2;20	80	260	86;40	43;20	60
Mercury	28	9;20	4;40	220	132	44	22	20
Sun	14	4;40	2;20	80				

[1]) These units are, of course, not "degrees" since every circle, independent of size, contains 360°. Here, however, we are dealing with arc-lengths measured in units of length $\frac{\pi}{180} R$ such that the circumference of a circle of radius smaller than R contains less than 360 units.

[2]) For the Sun cf. IX,7–8 (above p. 69 f.).

XVI,15–22. These verses give the rules for finding the true longitude λ of a planet from its mean longitude $\bar{\lambda}$, the latter assumed to be known, together with the mean longitude $\bar{\lambda}_0$ of the Sun and the parameters from Table 22.

The first step (XVI,15) consists in the determination of the equation σ caused by the epicyclic anomaly α (cf. Fig. 60). For an outer planet one has always

$$\alpha = \bar{\lambda}_0 - \bar{\lambda} \tag{1}$$

hence one can form with α as argument the trigonometric functions

$$\left.\begin{aligned} \text{bhuja (or bāhu)} &= \operatorname{Sin}\alpha = R \sin\alpha \\ \text{koṭi} &= \operatorname{Cos}\alpha = R\cos\alpha \end{aligned}\right\} \tag{2}$$

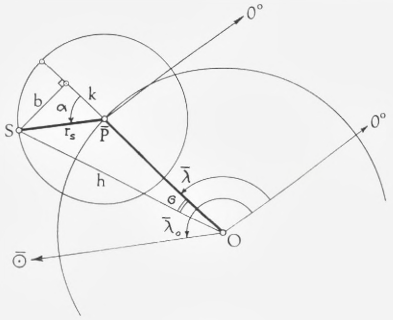

Fig. 60.

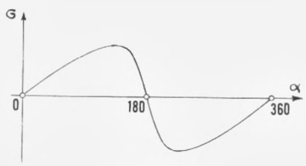

Fig. 61.

R being the radius of the deferent. Making use of the relation (1) of the preceding section one obtains (XVI,16)

$$\left.\begin{aligned} b &= \text{bhujaphala} = \frac{c_s}{360}\operatorname{Sin}\alpha = r_s \sin\alpha \\[2ex] k &= \text{koṭiphala} = \frac{c_s}{360}\operatorname{Cos}\alpha = r_s\cos\alpha \end{aligned}\right\} \tag{3}$$

(cf. Fig. 60).[1] Computing the "(śīghra-) hypotenuse"

$$OS = h = \sqrt{(R + r_s\cos\alpha)^2 + (r_s \sin\alpha)^2} \tag{4}$$

one finds (XVI,17)

$$\operatorname{Sin}\sigma = R\cdot\frac{b}{h}. \tag{5}$$

This gives the "śīghra-correction" σ; it is positive for $0 < \alpha < 180°$ and negative in the remaining semicircle. Fig. 61 shows the general type of the function $\sigma(\alpha)$.

[1]) The rules of signs in XVI,16 follow a terminology according to which "Aries 0°" means $\alpha = 0$. Consequently $k \geq 0$ for the arc "Capricorn 0° to Gemini 30°" and similarly for $k \leq 0$. Our notation $k = r_s \cos\alpha$ automatically takes care of these rules.

The next steps consist in a modification of the direction of the apsidal line, first under the influence of the śīghra correction, then by a manda correction (cf. Fig. 62).

The first displacement moves the apogee from A to A_1, and correspondingly the endpoint M of the manda-radius to M_1, by the amount of the angle $\frac{\sigma}{2}$ where σ is the śīghra correction found in (5). Having thus formed the "corrected longitude of the apogee" (XVI,18)

$$m_1 = m \mp \tfrac{1}{2}\sigma \tag{6}$$

one computes

$$\varkappa_1 = \bar{\lambda} - m_1 \tag{7}$$

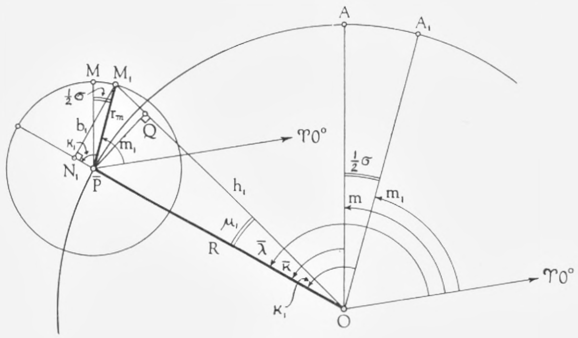

Fig. 62.

i.e., the angle made by the radius $r_m = M_1\bar{P}$ with the direction $O\bar{P}$. From k_1 one obtains, as in (3), the perpendicular $b_1 = M_1N_1$ from

$$\frac{c_m}{360}\,\mathrm{Sin}\,\varkappa_1 = r_m \sin\varkappa_1 = b_1 \tag{8}$$

which is seen from O under the angle μ_1.

In order to determine this angle μ_1 the same type of approximation is used as in IX,7–8 (above p. 69 f.) that is to say it is assumed (cf. Fig. 62) that

$$M_1N_1 = b_1 \approx \bar{P}Q = R\sin\mu_1$$

or

$$\mathrm{Sin}\,\mu_1 = b_1. \tag{9}$$

This manda correction is now used to correct once more the apsidal line by moving A_1 to A_2 (hence M_1 to M_2) by the angle $\frac{1}{2}\mu_1$ (cf. Fig. 63). Thus one forms

$$m_2 = m_1 \pm \mu_1 \tag{10}$$

and find from it (XVI,19)

$$\varkappa_2 = \bar{\lambda} - m_2. \tag{11}$$

Hence one can find the perpendicular $b_2 = \mathrm{M}_2\mathrm{N}_2$ from

$$\frac{c_{\mathrm{m}}}{360}\,\mathrm{Sin}\,\varkappa_2 = r_{\mathrm{m}} \sin \varkappa_2 = b_2 \tag{12}$$

to which corresponds a manda correction μ_2, again determined from

$$\mathrm{Sin}\,\mu_2 \approx b_2. \tag{13}$$

This correction μ_2 is now used to change the position of the center of the śīghra epicycle from the original mean position $\bar{\mathrm{P}}$ to a point $\bar{\mathrm{P}}_1$ of longitude $\bar{\lambda}_1$ such that (XVI,19)

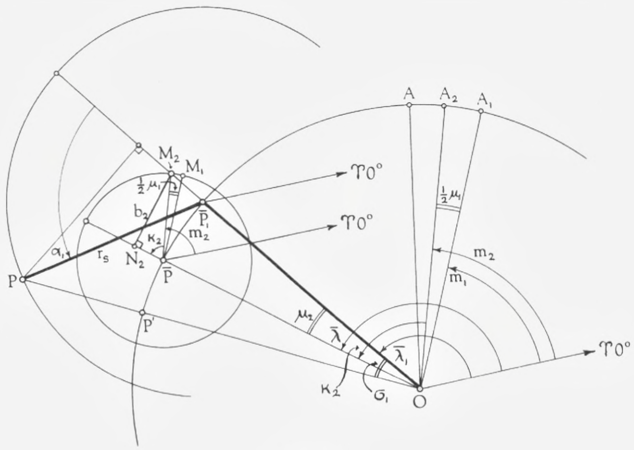

Fig. 63.

$$\bar{\lambda}_1 = \bar{\lambda} \mp \mu_2. \tag{14}$$

With $\bar{\mathrm{P}}_1$ as center one finds the śīghra correction σ_1 (of course by the same process as before σ from (1) to (5)) for the anomaly (XVI,20)

$$\alpha_1 = \bar{\lambda}_0 - \bar{\lambda}_1. \tag{15}$$

The resulting śīghra correction σ_1 then defines the true longitude of the planet from

$$\lambda = \bar{\lambda}_1 \pm \sigma_1 \tag{16}$$

which is in Fig. 63 the longitude of the point P′ on the deferent. This completes the computation of the true position of an outer planet.

For Venus and Mercury (XVI,21) the equation of center of the Sun is still to be taken into consideration. Let $\lambda_{\mathrm{A}0}$ be the (sidereally fixed) longitude of the solar apogee, c_0 the circumference of the manda epicycle of the Sun of radius r_0 (which is the equivalent of the solar eccentricity), $\bar{\lambda}_0$ the longitude of the mean Sun at the given moment and

$$\bar{\varkappa}_0 = \bar{\lambda}_0 - \lambda_{A0}. \tag{17}$$

Then

$$\frac{c_0}{360}\,\mathrm{Sin}\,\bar{\varkappa}_0 = r_0 \sin \bar{\varkappa}_0 \tag{18}$$

leads as before (e.g. (3) to (5)) to the manda correction μ_0 of the Sun. It gives us the amount by which the true Sun differs in longitude from the mean Sun; by the same amount the whole epicycle of an inner planet must be displaced with respect to the

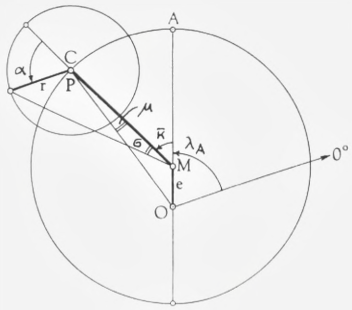

Fig. 64.

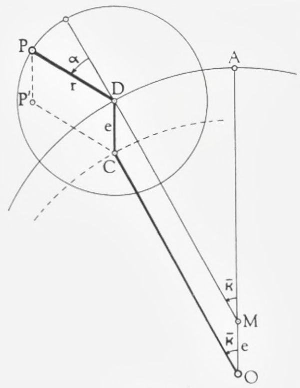

Fig. 65.

corrected mean position $\bar{\lambda}_1$ obtained in (14). Thus μ_0 has to be added to the value λ given by (16).

This solar correction μ_0 appears to be unique to the Pañcasiddhāntikā.[1]) An additional correction of $-1;7°$ is prescribed in XVI,22 for the longitude of Venus. This is probably an empirical correction, perhaps introduced by Varāhamihira.

With (16) we have finally the following rules for the determination of the true longitudes of the inner planets:

$$\left.\begin{array}{ll} \lambda = \bar{\lambda}_1 + \sigma_1 + \mu_0 - 1;7° & \text{for Venus} \\ \lambda = \bar{\lambda}_1 + \sigma_1 + \mu_0 & \text{for Mercury.} \end{array}\right\} \tag{19}$$

We now can turn to the problem of motivating the peculiar way in which the two equations, σ depending on α and μ depending on $\bar{\varkappa}$ (cf. Fig. 64), are combined in the rules from (6) to (16). Obviously the core of the difficulty of computing planetary positions lies in the fact that two inequalities are superimposed and that all combinations of the two effects are in principle possible. In other words we are dealing actually for a function of two independent variables which one tries to control, for

[1]) An attempt to correct for solar equation was made in Khaṇḍakhādyaka IX,9.

practical reasons, through a combination of values depending on each variable separately.

In order to explain the astronomical meaning of the compromise adopted by out text we replace the effect of the manda-epicycle by the equivalent eccentricity of the deferent and change our notation accordingly. Thus r_m becomes the eccentricity e of the deferent (cf. Fig. 64 and Fig. 65), while r_s is placed by r, the radius of the epicycle which carries the planet P. Without the effect of the eccentricity the mean planet would be at C and the true planet at P′ when subject to the anomaly α alone.

To combine the two displacements, one in the fixed direction CD in the amount e, the other CP′ depending on α, we assume that both are small in comparison to the

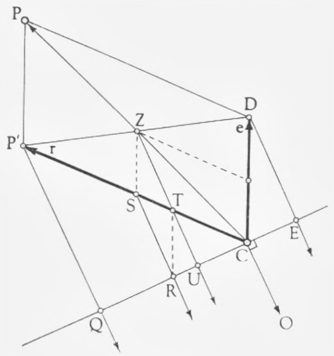

Fig. 66.

radius R of the deferent, such that the arc EQ of the deferent (cf. Fig. 66) may be considered a straight line, of course perpendicular to the direction OC. Under this assumption the observer at O would ascribe to the planet the longitude of Q when affected only by the śīghra correction $\sigma(\alpha) = \mathrm{CQ}$. Conversely the eccentricity e would move the center of the epicycle from C (which is at a distance $\bar{\varkappa}$ from the apogee of the deferent) to D, i.e., to the longitude of the point E. Hence CE is the manda correction $\mu(\bar{\varkappa})$.

According to the rules of the text neither $\sigma(\alpha)$ nor $\mu(\bar{\varkappa})$ are used directly. The point with the distance $\varkappa_1$ from the apogee, where

$$\varkappa_1 = \bar{\lambda} - m_1 = \bar{\varkappa} + \tfrac{1}{2}\sigma$$

(according to (6) and (7)) is represented by the midpoint R of CQ (cf. Fig. 66). The manda equation μ_1 which belongs to the point R would be obtained by projecting onto the deferent a vector from R parallel to CD and of length $e = \mathrm{CD}$. Formula (10), however, shows that only $\frac{1}{2}\mu$, is used, i.e., the projection RU of $\mathrm{RT} = \frac{1}{2}e$. The parallelograms drawn in Fig. 66 show that T is a point of CP′ and that U also can be obtained by projecting the midpoint Z of the parallelogram PP′CD onto the deferent. Thus the use of $\frac{1}{2}\sigma$ and $\frac{1}{2}\mu_1$ is shown to be the equivalent of introducing the midpoint of the resultant displacement of both inequalities, represented on the deferent by the point U.

One now considers U as the point for which the manda correction μ_2 should be obtained, then applied to C according to (14). This, in turn, results in some change in the epicyclic anomaly according to (15) and hence to a new śīghra correction σ_1, to be used as the final correction in (16).

The statement made above that U is the representative of the midpoint Z of the resultant inequality is no longer exact when QE is curved. Nevertheless the general idea remains valid that a point near the point Z on the resultant may produce a better correction than a correction provided by either D or P′ alone.

XVI,22. In the Paitāmahasiddhānta of the Viṣṇudharmottarapurāṇa IV,15 one finds the following rule which can explain what Varāhamihira had in mind:

"One should divide the difference between the corrected argument for the final śīghra operation and the argument for the first station by the difference of the (planet's) true velocity and the true velocity of the śīghra; the result is the time, in days and their parts, of the first station."

The idea underlying this rule can be summarized as follows. From III,31 in the same text is known the anomaly α_0 at which the first station should occur (and, symmetrically to 180°, the second station). Let α be the true anomaly found by computation for a moment t not too far ahead of the time for which the station may be expected. Hence $\alpha_0 - \alpha$ is the arc still to be travelled by the planet. Its velocity v on the epicycle is the difference between the velocity of the śīghra s and the mean velocity of the planet. The quotient $(\alpha_0 - \alpha)/v$ gives the time between t and the moment when the planet becomes stationary.

XVI,23. The following angular differences between Sun and planet are required for visibility:

Moon 12°	Jupiter 11°
Mars 17	Venus 9
Mercury 13	Saturn 15.

The same numbers are found in the Paitāmahasiddhānta of the Viṣṇudharmottarapurāṇa (III,9) and, e.g., in the Laghubhāskarīya (VII,1–2). In the latter text the degrees are transformed into vināḍikās by multiplication with 10, a procedure which shows that we are dealing with equatorial degrees, i.e., with arcs of oblique ascensions.

Similar, but slightly different data are given in XVII,58, excepting those for the Moon and Saturn that are unchanged.

XVI,24–25. These verses deal with the computation of the latitudes of the planets, but we do not understand how the given rules can lead to reasonable results. Latitudes should depend on three elements: the inclination of the planet's orbit (i.e. inclination of deferent and epicycle with respect to the ecliptic), the location of the nodes, and the distance of the planet from the earth (i.e., essentially, on the anomaly α). The last element is taken care of by the rule in XVI,25 to multiply a preliminary result by R/h where h is the corrected śīghra hypotenuse, i.e. the distance of the planet from the observer corrected for both inequalities (in first approximation shown by OS in Fig. 60 p. 103).

The two first mentioned effects should somehow appear in the rules which seem to involve the two planetary inequalities but the details remain obscure. The pairs of coefficients

Saturn	9/8	9/8
Jupiter	9/8	3/4
Mars	3/4	3/4
Venus	9/8	3/4
Mercury	3/4	9/8

could perhaps suggest a tilting of the epicycles about two orthogonal diameters but the restriction to only two numerical values, 3/4 = 0;45 and 9/8 = 1;7,30, makes any coordination with the individual conditions still more difficult to understand.

Chapter XVII

XVII,1–60. These sixty verses of the last chapter of the Pañcasiddhāntikā form a unit, clearly distinct from the preceding chapters. We find here a theory of the planetary motions and of the planetary phases in a form which is directly related to Babylonian methods, in marked contrast to the geometric models in the classical Greek fashion as found, e.g., in chapter XVI.

In the present text the planets are discussed one by one in the order

Venus Jupiter Saturn Mars Mercury.

This arrangement is very unusual since it differs from the ordinary sequence in Indian astronomy (e.g. in XVII,65–80)

Mars Mercury Jupiter Venus Saturn

which is the sequence of the days of the week and thus ultimately based on the Hellenistic order

Saturn Jupiter Mars Venus Mercury.

It also differs from the Babylonian order

Jupiter Venus Mercury Saturn Mars.

Astrologically it groups together the benefic planets (Venus and Jupiter) and the malefic (Saturn and Mars), leaving the neutral Mercury for last; but this may be a completely fortuitous circumstance.

In order to avoid repetitions we shall not follow in our discussion the text verse by verse. First we will combine all data about the synodic periods from which the mean motions are derived. Then we will take up, planet by planet (from Saturn to Mercury), problems concerning the planetary phases, their distribution and natural order. Finally epoch constants and visibility conditions will be treated, again in separate sections.

1. Synodic Periods

As "synodic period" or "synodic time" one denotes the time interval between one heliacal rising of a planet and the next, or, in general, from one phase to the next of the same kind. If one wants to establish the date of a certain phase one must know the number of synodic periods elapsed since the given epoch. The following procedure is designed to furnish this information.

Let a be the given ahargaṇa and a_0 a positive or negative correction of a which leads from the epoch date to the nearest phase—for which we use ordinarily the first visibility after conjunction (for an inner planet: after inferior conjunction) and which we denote by Γ. We wish to know the number s of synodic periods of a given length $\bar{p}$ contained in

$$a' = a + a_0 \tag{1}$$

days. In our text the obvious answer

$$\bar{s} = a'/\bar{p} \tag{2}$$

is not reached directly but only as a result of modifications of approximate values p and s where

$$s = a'/p. \tag{3}$$

If ζ is a correction which changes the approximate period p to the accurate period $\bar{p}$ by

$$p + \zeta = \bar{p} \tag{4}$$

one has

$$\bar{s} = \frac{a'}{\bar{p}} = \frac{a'}{p+\zeta} = \frac{s}{1+\zeta/p} \approx s\left(1 - \frac{\zeta}{p}\right) = \frac{a' - \zeta s}{p}. \tag{5}$$

Table 23 shows the values taken from the text for a_0, p, and ζ.

Let us assume that a certain phase (e.g. Γ) occurs at a point of longitude λ. The next occurrences will take place at about $\lambda + \overline{\Delta\lambda}$, $\lambda + 2\overline{\Delta\lambda}$, etc., where $\overline{\Delta\lambda}$ represents the "mean synodic arc". As the planet traverses the ecliptic the phases will be more or less equidistantly spaced, but one sidereal rotation will not result in an occurrence of the phase in question at the starting point λ. In general it will take Z rotations (or $Z + \Pi$ rotations for Venus and Mars) before an accurate return of the phases to the same longitudes takes place, i.e., only after Π occurrences during Z (or $Z + \Pi$) revo-

TABLE 23.

	a_0	p	ζ	$\bar{p}$	ch. XVII
♄	$-150\frac{1}{3}^d$	378 = 6,18^d	+1/10	6,18; 6^d	14 – 15
♃	– 34;34	399 = 6,39	–1/9	6,38;53,20	6 – 7
♂	–216;40	780 = 13, 0	–0;2,41	12,59;57,19	21 – 22
♀	–147	584 = 9,44	–1/11	9,43;54,32...	1 – 2
☿	+28;20	927/8 = 1,55;52,30	+0; 0,15	1,55;52,45	36 – 37

lutions the phases will be periodically repeated. Since neither a planet nor the Sun move with constant angular velocity the "true synodic arcs" will more or less differ from the mean $\overline{\Delta\lambda}$. Nevertheless Π occurrences will result in a periodic repetition of all phases since the apsidal lines can be considered fixed for comparatively long intervals of time.

For an outer planet the sum of Π and of the corresponding number Z of sidereal revolutions of the planet gives the length of the basic period expressed in years; for an inner planet the number of sidereal rotations itself represents the number of years. Specifically one has the following relations:

$$\begin{array}{l} \text{Saturn, Jupiter} \\ \quad\text{Mars} \\ \quad\text{Venus} \\ \quad\text{Mercury} \end{array} \qquad \Pi \text{ occur.} = \left\{ \begin{array}{l} Z \text{ sid. rot.} = (\Pi+Z) \text{ sid. years} \\ (\Pi+Z) \text{ sid. rot.} = (2\Pi+Z) \text{ sid. years} \\ (\Pi+Z) \text{ sid. rot.} = (\Pi+Z) \text{ sid. years} \\ Z \text{ sid. rot.} = Z \text{ sid. years} \end{array} \right\} \qquad (6)$$

The quotient

$$P = \Pi/Z \qquad (7)$$

indicates the number of synodic intervals which correspond in the mean to one sidereal revolution of the planet. Consequently

$$\overline{\Delta\lambda} = \frac{360°}{P} = \frac{360° \cdot Z}{\Pi} \qquad (8)$$

gives the length of the mean synodic arc. In general P is, of course, not an integer.

Furthermore: if one divides the number $\bar{s}$ of synodic intervals contained in the given ahargaṇa a', found in (5), by the number P

$$\frac{\bar{s}}{P} = N \qquad (9)$$

then the result N tells us how many revolutions of the planetary phase under consideration (e.g., Γ) took place during a' days. Therefore the integer part of N can be ignored while the remainder gives the fraction of a revolution gained in longitude over the position at a_0. Hence (9) allows us to determine the mean longitude of the planet at the given moment.

TABLE 24.

	Babylonian			Pañcasiddhāntikā			
	Π	Z	$\overline{\Delta\lambda}$	$P = \Pi/Z$	$\overline{\Delta\lambda}$	ch. XVII	
♄	4,16	9	≈12;39°	256/ 9 = 4,16/ 9 = 28;26,40		15	♄
♃	6,31	36	≈33; 9	391/36 = 6,31/36 = 10;51,40		7–8	♃
♂	2,13	18	≈48;43	133/18 = 2,13/18 = 7;23,20		22–24	♂
♀	12, 0	7,11	3,35;30		3,35;50°	1	♀
☿	11,24	3,37	≈1,54;13	684/217 = 11,24/3,37 = 3;9,7,32,...		37–38	☿

The parameters (7) and (8) are at the foundation of Babylonian planetary theory. The close resemblance of the data recorded in XVII,1–60 to Babylonian procedures suggests a comparison with the above mentioned basic parameters. Table 24 reveals exact numerical agreement for the outer planets and Mercury.[1]) For Venus one finds a minor deviation but this is not to be taken too seriously since our knowledge of Babylonian data is particularly incomplete for this planet.

2. Patterns of Distribution for the Phases on the Ecliptic

If consecutive occurrences of a given phase were always spaced at a distance $\overline{\Delta\lambda}$ from each other the Π points on the ecliptic where our phase occurs would be also equidistantly arranged at a distance of

$$360°/\Pi = 360°/PZ = \overline{\Delta\lambda}/Z.$$

In fact, however, the length of the synodic arcs is greater in some sections of the ecliptic and therefore smaller in others, due to the anomaly of Sun and planet and other causes, such that the density of the Π points at which a certain phase will be observed depends on the region of the ecliptic.

To take care of this empirical fact Babylonian astronomy has invented several devices among which "System A" is of interest in the present context. The ecliptic is divided in a number of sections (from two to six are attested), generally of unequal length; within each of these sections the phases are equidistantly spaced, in some sections narrower, in others wider, than the mean distance $\overline{\Delta\lambda}/Z$. It is exactly this idea which we find applied in the present chapter.

Saturn **(XVII,16–19)**

The ecliptic is divided into three sections

(1)	length: $\alpha_1 =$	45;51°	containing 30 occurrences
(2)	$\alpha_2 =$	177;34	127
(3)	$\alpha_3 =$	136;35	99
Total		360;0	$\Pi = 256$

[1]) For the Babylonian parameters cf. ACT II p. 283.

We are not told, however, where these three arcs should be located on the ecliptic.

The lengths of these three sections α_1, α_2, α_3, are only little different from the mean lengths $\bar{\alpha}_1$, $\bar{\alpha}_2$, $\bar{\alpha}_3$, which one obtains from the mean distance

$$360°/\Pi = 6{,}0°/4{,}16 = 1;24{,}22{,}30°$$

by multiplication with the number of occurrences each arc contains. In this way one finds

$$\begin{aligned}\bar{\alpha}_1 &= 42;11{,}15° = \alpha_1 - 3;39{,}45°\\ \bar{\alpha}_2 &= 178;35{,}37{,}30 = \alpha_2 + 1;1{,}37{,}30\\ \bar{\alpha}_3 &= 139;13{,}7{,}30 = \alpha_3 + 2;38{,}7{,}30.\end{aligned}$$

This shows that the "true" distribution of the phases is only insignificantly different from the "mean" distribution and it is difficult to see why at all one should have introduced separate zones for such minute deviations.

Jupiter **(XVII,9–11)**

The ecliptic is divided into three sections

(1)	length: $\alpha_1 = 159;30°$	containing	180 occurrences
(2)	$\alpha_2 = 180;0$		195
(3)	$\alpha_3 = 20;30$		16
Total	360;0		$\Pi = 391$

The location of these sections on the ecliptic is not specified.

The length of the mean distance on the ecliptic between phases of the same kind is given by

$$360°/\Pi = 6{,}0°/6{,}31 \approx 0;55{,}14{,}34{,}40{,}49,\ldots°$$

Multiplication of this arc by the above given number of events gives mean arcs which are hardly different from the given arcs:

$$\begin{aligned}\bar{\alpha}_1 &= 165;43{,}44,\ldots° = \alpha_1 + 6;13{,}44,\ldots°\\ \bar{\alpha}_2 &= 179;32{,}22,\ldots = \alpha_2 - 0;27{,}37,\ldots\\ \bar{\alpha}_3 &= 14;43{,}53,\ldots = \alpha_3 - 5;46{,}7,\ldots\end{aligned}$$

As in the case of Saturn the true density of the occurrences is only little different from the mean one.

Mars

For a sixfold division of the ecliptic cf. below p. 119.

Venus (**XVII,1–2**)

The uniformity in the motion of Venus makes it unnecessary to introduce a sectioning of the ecliptic, a conclusion also known from Babylonian astronomy.

In XVII,1 the mean synodic arc is given as

$$\overline{\Delta\lambda} = 215;50°$$

(cf. Table 24), the mean synodic time as

$$\overline{\Delta t} = 583;54,32,\ldots^{d}$$

(cf. Table 23), i.e. about 593;11,... tithis. The corresponding Babylonian parameters are $\overline{\Delta\lambda} = 215;30°$ and $\overline{\Delta\tau} = 593;10^{\tau}$ respectively. Exactly the Babylonian value $\overline{\Delta\lambda} = 215;30°$ is also attested in our present text (XVII,75; cf. Table 33, below p. 126).

For the subdivision of the synodic arc cf. below p. 120 f.

Mercury (**XVII,38–40**)

For this planet we find here an eightfold division of the ecliptic. The individual arcs are

$$\begin{array}{llll} \alpha_1 = & 8° & \alpha_5 = & 12° \\ \alpha_2 = & 30 & \alpha_6 = & 30 \\ \alpha_3 = & 60 & \alpha_7 = & 97 \\ \alpha_4 = & 100 & \alpha_8 = & 23, \end{array}$$

correctly totalling 360°.

Associated with these segments are eight numbers as follows:

$$\begin{array}{llll} n_1 = & 7 & n_5 = & 14 \\ n_2 = & 30 & n_6 = & 33 \\ n_3 = & 81 & n_7 = & 104 \\ n_4 = & 88 & n_8 = & 31. \end{array}$$

One would expect these numbers to represent occurrences of a certain phase, presumably Ξ, within each segment, the total being Π. In fact, however, the total 388 = 6,28 is not attested as a number Π (and the corresponding $Z = \Pi/P$ with $P \approx 3;9,7$ would not be an integer) but as the number Z of years which contain Π = 20,23 occurrences.[1])

The fact that the numbers n_i do not represent occurrences but their number divided by P may have to do with the determination of true and mean longitudes of the phases described in the next section. The text calls the numbers n_i "days" which is in any case meaningless.

The mean distance between occurrences is given by $6,0°/\Pi$ ($\approx 0;17,39,\ldots$). An arc of length α_i should therefore contain in the mean $m_i = \alpha_i\Pi/6,0$ occurrences. If the n_i whose total is $Z = \Pi/P$ would correspond to a mean density their number

[1]) Cf. ACT II, p. 283.

on α_i should be $\bar{n}_i = \alpha_i Z/6,0$, hence $m_i/\bar{n}_i = \Pi/Z = P$. In fact the quotients $p_i = m_i/n_i$ are not constant. One finds

$$\begin{array}{ll} p_1 = 3;52,57,\ldots & p_5 = 2;54,42,\ldots \\ p_2 = 3;23,50 & p_6 = 3;5,18,\ldots \\ p_3 = 2;30,59,\ldots & p_7 = 3;10,6,\ldots \\ p_4 = 3;51,40 & p_8 = 2;31,13,\ldots \end{array}$$

But the variation from the mean value $P \approx 3;9,7$ are not very great, a fact reminiscent of the experience with Saturn and Jupiter.

3. Mean and True Positions

For Venus no distinction seems to be made between mean and true positions (cf. above p. 114). For the other planets the number N of its "risings" is to be multiplied by a certain number Z and the product divided by another number Π. These numbers Z and Π are the well known Babylonian parameters which determine the ratio $P = \Pi/Z$ (cf. above p. 111 and Table 24) that counts the number of mean synodic arcs (and their fractions) which cover exactly 360° in longitude. Hence we can give the above rule the form: the number of risings should be divided by P.

It is not difficult to find the reason for this operation. Let N be the number of "risings" i.e. the number of occurrences of the phase Γ of a planet since a first Γ after epoch. Suppose N is an integer multiple of P. Then we know that N synodic arcs cover an integer multiple of 360°; hence the first and the last rising have the same longitude. Consequently it is only the remainder in the quotient N/P which is of interest and which tells us which fraction $n < 1$ of P synodic arcs goes beyond the longitude of the first rising. The corresponding gain in mean longitude will be $nP\overline{\Delta\lambda}$.

The longitudinal progress will be greater than this amount in case the true synodic arcs $\Delta\lambda$ are greater than $\overline{\Delta\lambda}$, smaller for $\Delta\lambda < \overline{\Delta\lambda}$. This seems to be the meaning of the statements found in XVII,8, XVII,24, and XVII,41. That the correction is applied to "days" instead of to longitudes is a common mistake in this chapter (cf. p. 126 and p. 128). The transformation to corresponding time intervals would not be difficult since the mean synodic times are known ($\bar{p}$ in Table 23 p. 111). The information concerning the true synodic arcs would have to come from the schemes for the distribution of the occurrences of the phases in different sections of the ecliptic (cf. above p. 112).

So far the principle of the procedure seems clear. There are, however, additional steps mentioned in the text which we cannot properly explain.

Saturn (XVII,15–17) *and Jupiter* (XVII,7,9–10)

For both planets additive constants, +89 (in XVII,15) and +18 (in XVII,7) respectively, are mentioned which seem to be epoch constants. Their discussion is therefore postponed to a later section (p. 123–125).

Other positive and negative corrections are related to the remainders n (called "padas") of the quotients N/P. These corrections are

for Saturn	in	(1) 30 padas	$+2416'$	$= +40;16°$
(XVII,16–17)		(2) 127	$-2519'$	$= -41;59°$
		(3) 99	$+2037'$	$= +33;57°$
		total: $\Pi = 256$		

and similarly

for Jupiter	in	(1) 180 padas	$-1456'$	$= -24;16°$
(XVII,9–10)		(2) 195	$+1265'$	$= +21;5°$
		(3) 16	$-1391'$	$= -24;46°$
		total: $\Pi = 391$.		

If one takes the number of padas in which positive corrections are prescribed and compares it with the number of padas of opposite sign one finds

for Saturn: 129 positive, 127 negative
for Jupiter: 195 positive, 196 negative.

In the case of Jupiter 195 and 196 are the integers nearest to $\frac{1}{2}\Pi$.
In the case of Saturn one has $\frac{1}{2}\Pi \pm 1$ instead of simply $\frac{1}{2}\Pi = 128$.

The reason for this arrangement we do not know. On the values of the corrections it seems without influence.

The above given corrections are not final, either for Saturn or for Jupiter. For Saturn XVII,17 prescribes "a subtraction or addition" of 12;12° which perhaps should be applied as follows:

in	(1)	$+40;16° - 12;12°$	$=$	$+28;4°$
	(2)	$-41;59 + 12;12$	$=$	$-29;47$
	(3)	$+33;57 - 12;12$	$=$	$+21;45$.

Finally all numbers (i.e. 28;4 etc.) should be multiplied by a factor $31/32 = 0;58,7,30$.

For Jupiter no further corrections are prescribed save a reduction (in XVII,10) of all numbers by $5/8 = 0;37,40$. We have no explanation to offer for any of these numbers. Also the concluding remark that Jupiter "rises in the east (Γ) in so many minutes (of arc)" makes no sense to us.

Mercury **(XVII,36–37)**

Between the operations which fit the pattern of Table 23 and 24 we find in XVII,36 a subtraction of 1/8 of a day and a division of the number of risings by 4. We cannot explain these steps which seem in excess of the normal procedure.

4. Subdivision of the Synodic Arc

We denote the planetary phases by Greek letters. For an outer planet we have the following sequence:

Γ heliacal rising
Φ first station
Θ opposition
Ψ second station
Ω acronychal setting.

From Ω to Γ the planet is invisible. The longitudinal progress from one Γ to the next is the "synodic arc", the time elapsed during this motion is the "synodic time". In principle the same applies to the motion from Ω to Ω or to any other phase but the corresponding intervals need not to be the same for all phases. Only for the mean synodic arc $\overline{\Delta\lambda}$ could any pair of Greek letters be used.

For an inner planet we define

$$\left.\begin{array}{ll}\Gamma & \text{morning rising}\\ \Phi & \text{first station}\\ \Sigma & \text{morning setting}\end{array}\right\}\ \text{morning star}$$

invisible at superior conjunction

$$\left.\begin{array}{ll}\Xi & \text{evening rising}\\ \Psi & \text{second station}\\ \Omega & \text{evening setting}\end{array}\right\}\ \text{evening star}$$

invisible at inferior conjunction

Γ morning rising

In the preceding sections we collected the information concerning (a) the mean values obtainable from the periodicity of the phases and (b) the variations in density of a given phase (Γ) on the ecliptic within the total of Π occurrences. In both cases one is dealing only with one specific phase, independent of its relation to the other phases. What remains are data which describe the sequence of all phases within one synodic arc (e.g. from Γ to Γ), both with respect to motion in longitude and to time intervals. Since no mention is made of differences in length of the synodic arcs depending on different sections of the ecliptic the total of intervals between consecutive phases should be the mean value of the synodic arc and the synodic time. Such a pattern will be fairly close to the actually observable intervals and discrepancies can be absorbed by the stretch of invisibility from Ω to Γ. Consequently such a scheme is not meant to be extended beyond one synodic period, or rather it begins with Γ and ends at Ω.

Larger intervals between consecutive phases (e.g., between Γ and Φ) are sometimes subdivided into shorter sections. We denote the dividing points—which do not correspond to real planetary phases—by Greek letters with accents.

The striking parallelism with Babylonian data established in the preceding sections extends also into the present problems. Without going into greater detail we shall therefore add to the Indian analysis of the sequence of the planetary phases also a description of similar schemes found in Babylonian sources. That such a comparison is at all fruitful in spite of the only fragmentary character of our knowledge of Babylonian astronomy demonstrates how intimate the relation of early Indian astronomy and its Babylonian predecessor must have been.

Saturn **(XVII,19–20)**

Table 25 shows the given data. For the Babylonian theory we have only a pattern with two velocity zones[1]) which divide the direct motion in a different fashion. Hence we have here no means for a closer comparison.

TABLE 25.

Saturn	ch. XVII,19–20		Babylonian	
$\Gamma \to \Gamma'$	16^d	1;20°		
$\Gamma' \to \Phi$	56	3;52		
$\Phi \to \Theta$	55	−3	$52;30^d$	−8° or −6;40°
$\Theta \to \Psi$	60	−4	60	
$\Psi \to \Omega'$	112	8		
$\Omega' \to \Omega$	36	3		
$\Gamma \to \Omega$	335^d	9;12°		

Since XVII,14–15 give for the mean synodic time $\bar{p} \approx 378^d$ (cf. Table 23 p. 111) one may estimate the time of invisibility $\Omega \to \Gamma$ as about 43^d with about 3;30° of direct motion since the mean synodic arc should be about 12;40° (cf. Table 24 p. 112).

Jupiter **(XVII,12–13)**

For this planet one finds (cf. Table 26) good agreement with the previously determined parameters (cf. Table 23 p. 111): $\bar{p} = 398;53,20^d \approx 399^d$ and $\overline{\Delta\lambda} = 33;9° \approx 34°$ (cf. Table 24). A corresponding Babylonian pattern is also shown in Table 26.

TABLE 26.

Jupiter	ch. XVII,12–13		Babylonian		
$\Gamma \to \Gamma'$	60^d	12°	$\Gamma \to \Gamma'$	30^d	7; 2°
$\Gamma' \to \Gamma''$.....	40	4	Γ'		
$\Gamma'' \to \Phi$	24	2	Φ	90	11;15
$\Phi \to \Theta$	56	−6	Φ		
$\Theta \to \Psi$	60	−6	Ψ	120	−9;23
$\Psi \to \Omega'$	80	12	$\Psi \to \Omega'$	90	10;47
$\Omega' \to \Omega$	50	9	$\Omega' \to \Omega$	30	7; 2
$\Omega \to \Gamma$	29	7	$[\Omega \to \Gamma]$	[30]	7; 2
$\Gamma \to \Gamma$	399^d	34°	$\Gamma \to \Gamma$	390^d	33;45°

[1]) Cf. ACT II p. 315.

Mars **(XVII,25–35)**

These eleven verses are meant to give a description of the subdivision of the synodic arc of Mars into five sections (gatis), thrown into confusion by the insertion of a detailed description of the retrograde motion.

At the beginning (XVII,25–26) we have the following arcs between the consecutive phases (C denoting the conjunction with the Sun):

(I)	$\Gamma \rightarrow \Phi$	186°
(II)	$\Phi \rightarrow \Psi$	−18
(III)	$\Psi \rightarrow \Omega$	180
(IV)	$\Omega \rightarrow$ C	30
(V)	C $\rightarrow \Gamma$	30
Total:	$\Gamma \rightarrow \Gamma$	408° ≡ 48° (mod. 360°).

A final synodic motion of 48° agrees well with the data found in XVII,22 (cf. Table 24). The text obviously considers as point of separation between gatis (II) and (III) the opposition Θ by describing it as "half of its course since conjunction" (XVII,25). This, however, must be a mistake since 18° can only be the total arc of retrogradation. Furthermore, by adding (V), (I), and (II) one finds 198° instead of 204° whereas $\Phi \rightarrow \Theta = -9°$ would lead to $C \rightarrow \Theta = 207°$.

The intrusion which concerns retrogradation consists of the verses XVII,29–33. Since this material is divided into three sections Varāhamihira (or his source) took this as representing 3 gatis; since XVII,27 and 28 deal with the gatis (I) $\Gamma \rightarrow \Phi$ and (II) $\Phi \rightarrow \Theta$ he counted XVII,34 as the 6th gati, concerning $\Psi \rightarrow \Omega$, the "fast gati" after retrogradation, while XVII,35 for $\Omega \rightarrow$C and C$\rightarrow \Gamma$ become gatis "7" and "8".

Excluding the foreign material XVII,29–33 we obtain the following pattern for the time intervals:

XVII,27:	(I)	$\Gamma \rightarrow \Phi$	267;30^d
XVII,28:	(II)	$\Phi \rightarrow \Psi$	between 51^d and 72^d
XVII,34:	(III)	$\Psi \rightarrow \Omega$	between 296^d and 314^d
XVII,35:	(IV)	$\Omega \rightarrow$ C	between 60^d and 72^d
	(V)	C $\rightarrow \Gamma$	between 60^d and 72^d.

The total which may vary between 735^d and 798^d (mean value 767^d) agrees reasonably well with the mean synodic time of about 780^d. Table 27 shows the distribution of the intervals within the zodiacal signs. The pairing Pisces and Aries, Taurus and Gemini, etc., is a characteristic feature of the Babylonian theory of Mars[1]) and assures us of a Babylonian archetype of the present material.

We now turn to the verses XVII,29–33 which concern the retrogradations of Mars, again arranged for pairs of zodiacal signs. In order to bring some sense in these data we have to assume another mistake. For each pair of signs we find three arcs

[1]) Cf. ACT II p. 302–306.

TABLE 27.

Mars	♓ ♈	♉ ♊	♋ ♌	♍ ♎	♏ ♐	♑ ♒	XVII
I ...	267;30^d						27
II ...	57^d	71^d	72^d	66^d	61^d	51^d	28
III ...	301 305	308 311	314 311	309 306	302 299	296	34
IV ...	62	69	72	69	63	60	35
V ...	62	69	72	69	63	60	
Total .	750 754	785 788	798 795	781 778	757 754	735	

and three corresponding time intervals. We assume that the first two concern $\Phi\to\Theta$ and $\Theta\to\Psi$ respectively (cf. Table 28) whereas the third belongs to an independent scheme for $\Phi\to\Psi$ (cf. Table 29), erroneously combined with the first two. This latter scheme not only forms a typical linear zigzag function for arcs as well as for days but the arcs agree exactly with the Babylonian "Scheme R" for the retrogradations of Mars.[1]) The data shown in Table 28 show the same pattern for the arcs $\Phi\to\Psi$. The time intervals, however, look rather garbled and we do not dare to offer any emendations.

What XVII,33 is intended to express escapes us, except for the final statement which seems to indicate that the second part of the retrograde arc, i.e., $\Theta\to\Psi$, should be 4/3 of the first part ($\Phi\to\Theta$). In the Babylonian theory the corresponding factor is 3/2.

Venus (XVII,3–5)

The only clear section concerns the direct motion of Venus as evening star from Ξ to Ψ with slowly decreasing velocity as one approaches the stationary point:

TABLE 28.

Mars	♑, ♒	♓, ♈ and ♏, ♐	♉, ♊ and ♍, ♎	♋, ♌
$\Phi\to\Theta$	– 6° 32^d	– 6° 42^d	– 7° 40^d	– 7° 44^d
$\Theta\to\Psi$	– 9° 39^d	–10° 42^d	–10° 40^d	–11° 40^d
$\Phi\to\Psi$	–15° 71^d	–16° 84^d	–17° 80^d	–18° 84^d

TABLE 29.

Mars $\Phi\to\Psi$	♑, ♒	♓, ♈ and ♏, ♐	♉, ♊ and ♍, ♎	♋, ♌
XVII,29–32	–15° 57^d	–16° 60^d	–17° 63^d	–18° 66^d
Babyl. "R"	–15°	–16°	–17°	–18°

[1]) Cf. ACT II p. 305 f. and Fig. 58 a.

Ξ	60^d	$74°$
$\downarrow$	60	73
	60	72
	27;30	20
Ψ	3	1;15

giving a total of 210;30^d and 240;15° in direct motion.

The retrograde motion near inferior conjunction is divided as follows:

Ψ	15^d	$-2°$
Ω	5	$-2(?)$
$\Omega \to \Gamma$	10	$-4(?)$
$\Gamma \to \Phi$	20	-4

i.e. lasting 50^d, perhaps with a total of 12° in retrograde motion.

One might assume that the direct motion as morning star (i.e. $\Phi \to \Sigma$) is of the same amount as the direct motion as evening star (i.e. $\Xi \to \Psi$). This would result in a total

$$\Xi \to \Sigma \qquad 471^d \qquad 468;30°.$$

According to Table 23 p. 111 the synodic period amounts to about 584^d with a longitudinal motion of 575;50° (cf. Table 24 p. 112). This would give for the period of invisibility at superior conjunction

$$[\Sigma \to \Xi] \qquad 113^d \qquad 107°.$$

This relation cannot be correct, however, since during this period the planet must move with more than solar velocity (cf. also the above given pattern for $\Xi \to \Psi$). This seems also to be expressed in the last sentence of XVII,5:

$$\Sigma \to \Xi \qquad 60^d \qquad 75°$$

which is exactly what one should expect. Consequently this would lead to

$$[\Phi \to \Sigma] \qquad 264;30^d \qquad 272;15°$$

or 54^d and 32° more than for $\Xi \to \Psi$, a rather implausible conclusion.[1] At any event the description of the motion of Venus as given in our text seems incomplete.

Mercury **(XVII,42–56)**

We have here a group of verses which deal with four separate cases in a perfectly parallel fashion (cf. Table 30 and Figs. 67 and 68). In each case we are given for the single zodiacal signs from Aries to Pisces an amount T of days and an amount

[1]) We have an early Babylonian text (5th century B.C.) which assumes for $\Phi \to \Sigma$ a motion of about 248° in 227^τ in good agreement with $\Xi \to \Psi$ in the present text; cf. Neugebauer-Sachs [1967] p. 197 (cf. below p. 128).

TABLE 30.

☿	$\Xi \to \Omega$		$\Omega \to \Gamma$		$\Gamma \to \Sigma$		$\Sigma \to \Xi$		Totals		☿
	T	B	T	B	T	B	T	B	T	B	
♈	36^d	35°	25^d	22°	29^d	21°	29^d	54°	119^d	88°	♈
♉	45	44	23	17	23	23	49	69	140	119	♉
♊	45	48	20	14	26	27	47	75	138	136	♊
♋	42	43	18	9	30	31	46	71	136	136	♋
♌	34	34	16	9	32	32	45	70	127	127	♌
♍	26	27	18	9	33	35	43	70	120	123	♍
♎	21	18	20	12	35	36	40	70	116	112	♎
♏	16	14	25	18	44	43	38	64	123	103	♏
♐	16	15	26	21	42	43	32	62	120	99	♐
♑	20	19	27	28	38	39	32	58	117	88	♑
♒	23	22	26	25	35	33	35	60	119	90	♒
♓	24	24	25	24	29	24	27	49	105	73	♓

B of degrees (which we call "pushes" in time and longitude respectively), leading from one phase to the next:

I: $\Xi \to \Omega$
II: $\Omega \to \Gamma$
III: $\Gamma \to \Sigma$
IV: $\Sigma \to \Xi$

The whole pattern has close relations to the Babylonian theory of Mercury[1]) which operates with similar "pushes", although in the texts known to us only for the

TABLE 31.

☿	$\Xi \to \Omega$		$\Gamma \to \Sigma$		☿
	T	B	T	B	
♈	36^d	36°	14^d	12°	♈
♉	42	42	16	14	♉
♊	48	46	19	18	♊
♋	44	42	24	22	♋
♌	38	36	27	26	♌
♍	22	22	30	30	♍
♎	15	14	36	34	♎
♏	15	14	46	44	♏
♐	16	16	46	44	♐
♑	22	20	44	42	♑
♒	24	22	34	34	♒
♓	24	22	24	24	♓

[1]) Cf. ACT II p. 293–295.

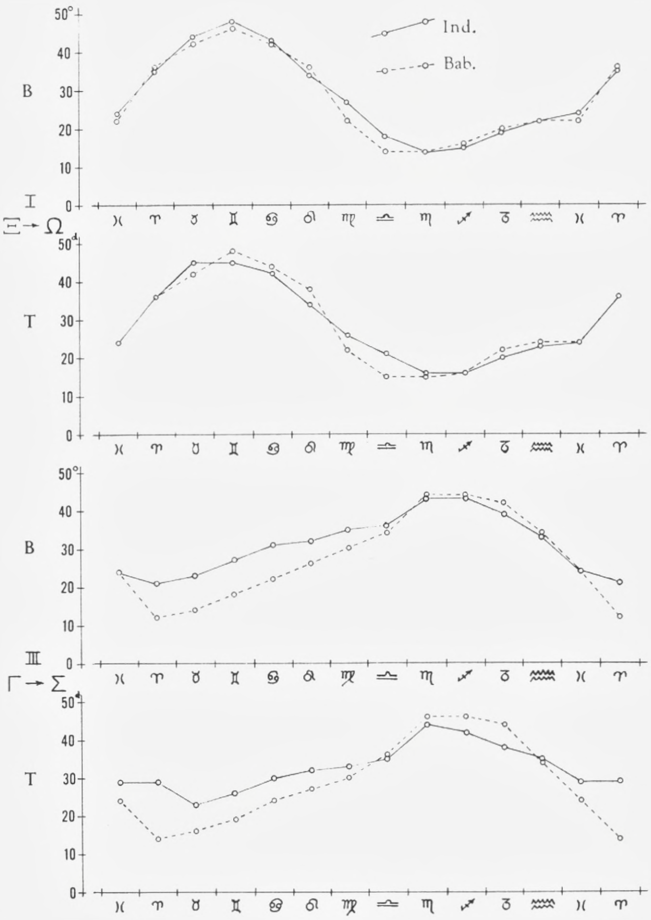

Fig. 67.

visible sections of the orbit, i.e., for $\Xi \to \Omega$ (here case I) and $\Gamma \to \Sigma$ (case III) as shown in Table 31 and Fig. 67. The numerical agreement is not perfect but the parallelism in the general trend and in the method itself is obvious.

We cannot connect any rational procedure with the verses XVII,54–56.

5. Epoch Values

In Table 23 (p. 111) we have a list of numbers a_0, representing days, which are elapsed between the epoch date and the nearby date of a specific planetary phase, i.e. heliacal rising (Γ) for the outer planets, first appearance as evening star (Ξ) for Venus and Mercury.

Assuming A.D. 505 March 22 as the date of the epoch (cf. pt. I, p. 8) a date a_0 days later (algebraically) should give us the date of the phase. Modern tables provide us with the corresponding elongations from the Sun. As Table 32 shows these elon-

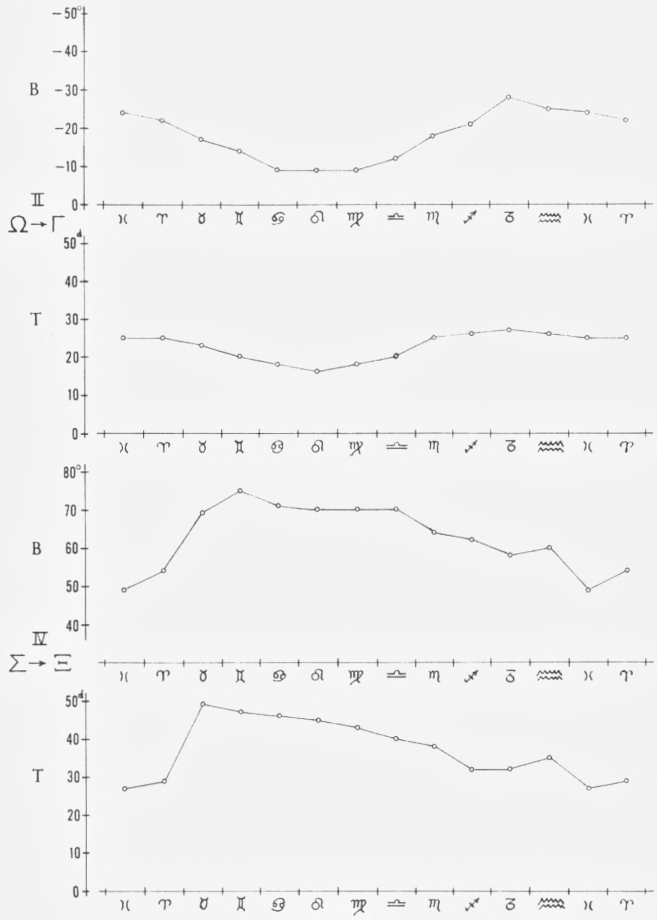

Fig. 68.

gations agree reasonably well with the visibility limits $\Delta\alpha$ given in XVII,58–60 (cf. below, p. 125). The only exception is Venus where a_0 leads only to the conjunction with the Sun. In XVII,2, however, we are given Virgo 26° as longitude of Ξ. This is indeed in agreement with the expected elongation since Venus reaches this position in A.D. 505 Sept. 10 when the Sun is about at Virgo 19°.

For Mercury we have no epoch constant giving us directly a longitude. For the outer planets we have in XVII,7 an additive constant of 18 (degrees) for Jupiter. Interpreting this as the planet's longitude at its first Γ after epoch we would have exact agreement with the position found for the date derived from XVII,6 (cf. Table 32). Consequently one should also interpret the corresponding constants for Saturn (in XVII,15) and for Mars (in XVII,23) as longitudes of the respective phases. For Mars this would mean a longitude of 175° (instead of 194° derived on the basis of a_0 in Table 32). This longitude would correspond to Sept. 27 and a solar position at 186°, hence to an elongation of 11°. For Saturn, however, no such agreement seems obtainable for the given numbers.

TABLE 32.

XVII		$-a_0$	505 March 22 $-a_0$	Planet λ	Sun λ_S	$\Delta\lambda = \lambda_S - \lambda$	$\Delta\alpha$ (XVII,58–60)
14	Saturn	$+150;20^d$	505 Aug. 19	135°	148°	13° Γ	15°
6	Jupiter	$+34;34$	Apr. 26	18	38	20 Γ	15
21	Mars	$+216;40$	Oct. 25	194	214	20 Γ	14
1	Venus	$+147$	Aug. 16	145	145	0	(−)8
36	Mercury	$-28;20$	Febr. 22	350	336	−14 Ξ	(−)12

6. First Visibility (XVII,57–60)

Of these four verses the second one (XVII,58) presents the most essential data, i.e. elongations from the Sun for each planet (and the Moon) required for visibility:

$$\left.\begin{array}{llll} \text{Saturn:} & 15^\circ & \text{in XVI,23:} & 15^\circ \\ \text{Jupiter:} & 15 & & 11 \\ \text{Mars:} & 14 & & 17 \\ \text{Venus:} & 8 & & 9 \\ \text{Mercury:} & 12 & & 13 \\ \text{Moon:} & 12 & & 12. \end{array}\right\} \quad (1)$$

Arranged according to permissible approach to the Sun, i.e. brightness, we would have here the order: Venus, Moon = Mercury, Mars, Saturn = Jupiter. In XVI,23 the arrangement would be: Venus, Jupiter, Moon, Mercury, Saturn, Mars.

In XVII,60 we are told that the arcs in question should be subtracted from the longitude of the Sun in the case of the planets but added for the Moon. This shows that the planets are assumed to be near the eastern horizon, i.e. the phase in question is always Γ, i.e. heliacal rising for the outer planets, first appearance as morning star for the inner planets. In the preceding sections it was always Ξ, i.e. first appearance as evening star, which played the leading role for Venus and Mercury.

The remaining verses, XVII,59 and XVII,57, seem to concern corrections to (1) depending on variable positions of the ecliptic with respect to the horizon. In XVII,59 it is said that the values in (1) should be multiplied by a corrective factor $c = \frac{30^\circ}{\varrho}$ where ϱ seems to mean the rising time of the zodiacal sign in question. If the ecliptic rises to the south of the eastpoint it is steeper to the horizon than the equator, hence $\varrho > 30^\circ$ and $c < 1$. Similarly $c > 1$ when the ecliptic rises to the north of the east-point. This implies that the values in (1) refer to arcs on the equator, as one would indeed expect in Indian astronomy (cf. also XVI,23). That $c < 1$ to the south of the east-point, $c > 1$ to the north is reasonable since the visibility of a planet depends essentially on the vertical distance of the Sun below the horizon (the so-called arcus visionis).

We meet again references to south and north (presumably from the east-point) in XVII,57, south having a subtractive, north an additive effect. If this correction would again apply to the values in (1) one would have here only another method for solving the same problem as in XVII,59. But all details escape us.

The number 480 could be 10 Sin ε with $\varepsilon \approx 24°$ and $R = 120$, modified by an unexplained factor 10. The term "latitude" could also mean declination but we see no reason for computing $\mathrm{Sin}\,\delta/10\ \mathrm{Sin}\,\varepsilon$.

XVII,64–80. In conclusion Varāhamihira has recorded a set of parameters, apparently from the Pauliśasiddhānta, concerning the mean synodic arcs of the planets and their subdivision by the planetary phases. The formulation in the text is marred by a misunderstanding. Varāhamihira calls ahargaṇa what is actually the planet's longitude (at a given date) and consequently takes the results obtained as "days" instead of degrees. The error originated probably in XVII,65 where degrees of solar motion and days are considered to be equivalent. Another consequence of this misunderstanding is probably the repeated division by the quantity which we here call b (e.g. $b = 4$ in XVII,66). We disregard these mistakes from now on in our commentary.

In order to avoid repetitions we regroup the different verses for our summary according to parallel subject matters. Since we are dealing again with methods which are closely related to ultimately Babylonian procedures we also give the relevant data from the cuneiform sources.

We begin with a group of rules asking for the computation of expressions of the form

$$\frac{(\lambda-\lambda_0)\,b}{a} = \frac{\lambda-\lambda_0}{\overline{\Delta\lambda}} \tag{1}$$

with given parameters λ_0,[1] a, and b, listed in our Table 33 (which also should be compared with Table 24 (p. 112). That

$$b/a = \overline{\Delta\lambda} \tag{2}$$

represents the mean synodic arc of the planet in question is evident from the numerical values. It is also clear that λ_0 represents degrees and minutes of arc; consequently

Table 33.

	ch. XVII	λ_0	a	b	$\overline{\Delta\lambda}$ = a/b	$\overline{\Delta\lambda}$ Babylonian	ACT p.
♄	78	16518′ = 275;18°	1118 = 18,38	3	6,12;40°	(6,0+)12;39,22,30	313
♃	72	1652′ = 27;32	2752 = 45,52	7	6,33; 8,34,...	(6,0+)33; 8,44,48,...	307
♂	66	6329′ = 105;29	3075 = 51,15	4	12,48;45	(6,0+)6,48;43,18,29,.	302
♀	75	11122′ = 185;22	1151 = 19,11	2	9,35;30	9,35;30	283
☿	69	1681′ = 28; 1	3312 = 52,12	29	1,54;12,24, ..	1,54;12,...	283

[1]) In XVII,75 the term $\lambda-\lambda_0$ is described as a subtraction from "the ahargaṇa" instead of from "the longitude of the planet." The parallelism of the corresponding sentences in XVII,66, 69, 72, and 78 leaves no doubt that the same mistake has to be assumed in all cases.

$\lambda - \lambda_0$ must be a longitudinal arc, reckoned from some initial position λ_0. If the division by $\overline{\Delta\lambda}$ results in an integer we know that the arc in question begins and ends with the same planetary phase (operating here with the mean distribution). If, however, (1) produces a fractional remainder then we know from it how far the planet is removed from the phase at the beginning of the synodic arc. As beginning may serve, of course, not only a phase like Γ but also, e.g., a (mean) conjunction with the Sun. We do not know, however, how the initial positions λ_0 were chosen. All planets, excepting Saturn, reach the longitude λ_0 during A.D. 505 but only Venus would then be near a characteristic phase (Ξ, about September 17; cf. also above p. 124).

The subdivision of the synodic arcs follows a common pattern as seen in our Table 34. Three steps lead from the conjunction (C) over heliacal rising (Γ) to the first station (Φ), then follows the retrograde arc (from Φ to Ψ) and finally again three steps from Ψ to setting (Ω) back to C. For these steps the motion of the Sun (in degrees or "days") is given and the corresponding increase of elongation of the planet. The total of these steps must be $\overline{\Delta\lambda}$ for the Sun and 360° for the elongations. Both conditions are very well satisfied for all three superior planets.

During the planet's direct motion the Sun moves faster than the elongation increases. For the retrograde arcs, however, the elongation exceeds the solar progress and the difference represents the length of the retrograde arc. Thus we find the following retrogradations

Saturn:	113° − 120° = − 7°	ACT p. 315 :	−6;40° and −8°
Jupiter:	109 − 120 = −11	p. 312 f.:	−8 to −10;12
Mars:	72 − 90 = −18	p. 305 f.:	−15 to −18;45

in good agreement with the Babylonian data. One should note that the midpoint of the retrograde arc leads in all cases to an elongation of exactly 180° which shows

TABLE 34.

	Saturn ch. XVII,78–80				Jupiter ch. XVII,73–74				Mars ch. XVII,67–68			
	solar motion		elongation		solar motion		elongation		solar motion		elongation	
C → Γ	18°		16;30°		16°		12°		36°		15°	
Γ	98		90;30		54		44		188		60	
Φ	14	130	13	120	70	140	64	120	108	332	60	135
$\Phi \to \Psi$	113		120		109		120		72		90	
Ψ	98		91		88		76		68		50	
Ω	13		12;30		40		32		240		70	
$\Omega \to$ C	19	130	16;30	120	16	144	12	120	56	364	15	135
Totals	373 = 6,13° $\overline{\Delta\lambda}$ = 6,12;40		360°		393 = 6,33° $\overline{\Delta\lambda}$ = 6,33;8,..		360°		768 = 12,48° $\overline{\Delta\lambda}$ = 12,48;45		360°	

TABLE 35.

ch. XVII 76 – 77	Venus			
	solar motion		elongation	
$C \to \Gamma$	5°		9°	
$\Gamma \to \Phi$	15	20°	21	−30
Φ	208		15	
Σ	12		5	
$\Sigma \to S$	48	268	10	+30
Totals	288° = 4,48° $\overline{\Delta\lambda}$ = 9,35;30°		0°	

that the opposition Θ is here assumed to divide the retrograde arc symmetrically —in contrast to the known Babylonian schemes for Mars (cf. above p. 120).

For Venus we have a slightly different pattern. The regularity of the motion of this planet makes it possible to consider only the motion from the inferior conjunction (C) to the superior conjunction (S), assuming that the other half of the motion, from S to C, is symmetric to the first half. Thus one obtains Table 35 which would give a total synodic arc of $2 \cdot 4{,}48° = 9{,}36°$ in excellent agreement with the required $\overline{\Delta\lambda} = 6{,}0° + 3{,}35;30°$.

For Mercury one finds Table 36, again with a correct total for the synodic motion. In the text the degrees of solar motion are schematically replaced by "days".

We do not possess for the inner planets similar Babylonian patterns for the subdivision of the synodic arcs but this is undoubtedly caused by the accidents of preservation.

TABLE 36.

ch. XVII 70 – 71	Mercury			
	solar motion		elongation	
$C \to \Gamma$	10°		12°	
$\Gamma \to \Phi$	14	24°	5	−17°
$\Phi \to \Sigma$	18		14	
$\Sigma \to \Xi$	30		6	
$\Xi \to \Psi$	18	66	14	34
$\Psi \to \Omega$	16		8	
$\Omega \to C$	8	24	9	−17
Totals	114° = 1,54° $\overline{\Delta\lambda}$ = 1,54;12,...		0°	

2. Short Terminological Glossary

For a complete index see Part I p. 187 ff.

ahargaṇa	number of days since epoch
ārdharātrika	system "midnight-system". for the characteristic parameters cf. I, 14 Table 1 (p. 12).
Avantī	see Ujjayinī
ayana	semicircle of the ecliptic, usually bounded by the solstices
bāhu	one side of a right triangle: $b = c \cos \alpha$
bhuja	same as bāhu
Caitra	name of a month (cf. Part I p. 187)
gati	arc between consecutive Greek-letter phenomena (i.e phases) of a planet
gola	semicircle of the ecliptic, bounded by the equinoxes
kakṣā	geocentric distance of a point on the orbit of a planet, etc.
karaṇa	1/2 tithi (cf. III, 19)
koṭi	one side of a right triangle: $a = c \sin \alpha$
kṛṣṇapakṣa	second half of the synodic month, following opposition (cf. III, 18)
kṣepa	epoch constant
mahāyuga	interval of 20,0,0,0 years
muhūrta	time interval of 1/30 day, i.e., 48 minutes
nāḍī	time interval of 1/60 day, i.e., 24 minutes
nakṣatra	"equal nakṣatra": arc of 13;20° of longitude, i.e., 360°/27 (cf. Part I p. 187)
rāhu	the ascending lunar node
Śaka	Śaka-era: year 1 beginning in A.D. 78
śuklapakṣa	first half of the synodic month (cf. III, 18)
tithi	1/30 of one synodic month
Ujjayinī	also called Avantī; locality at 75;50° east of Greenwich, $\varphi = 23;11°$
vināḍī	time interval of 1/60 nāḍī, thus 0;0,1 day = 24 seconds
Yavanapura	Alexandria in Egypt
yojana	unit of length such that 3200 yojanas = terrestrial equator
yuga	number of years, being a common period of several phenomena

3. Notation

It is, of course, impossible to associate letters and concepts in a strictly unique fashion. We nevertheless tried to adhere, within reasonable limits, to a consistent notation.

Trigonometry

$R\ldots$ radius of the basic circle and of the celestial sphere, usually $R = 120$, if not stated otherwise

$\mathrm{Sin}\,\alpha = R \sin\alpha, \quad \mathrm{Cos}\,\alpha = R\cos\alpha$

$\mathrm{Vers}\,\alpha = R - \mathrm{Cos}\,\alpha \qquad \mathrm{Crd}\,\alpha = 2\,\mathrm{Sin}\,\frac{\alpha}{2}$

Gnomon

$g\ldots$ length of vertical gnomon, usually $g = 12$

$s\ldots$ length of shadow of g, $s_0\ldots$ equinoctial noon shadow

$h_s = \sqrt{s^2+g^2}$ …"hypotenuse" to s

Spherical Astronomy

$\lambda\ldots$ longitude, usually sidereal, but also tropical, depending on context

$\beta\ldots$ latitude

$m\ldots$ "*mediatio*" (or "polar longitude"),

$b\ldots$ polar latitude

$\alpha\ldots$ right ascension, $\delta\ldots$ declination

$\varepsilon\ldots$ obliquity of ecliptic

Z… zenith, V… nonagesimal

$z\ldots$ zenith distance $h\ldots$ altitude, $\bar{h} = 90 - h$

$\varphi\ldots$ terrestrial (= geographical) latitude, $\bar{\varphi} = 90 - \varphi\ldots$ colatitude

$\eta\ldots$ ortive amplitude (also for setting amplitude)

$\omega\ldots$ ascensional difference

$e\ldots$ "earth Sine" $r\ldots$ "day radius" in sphere of radius R (cf. Fig. 13 p. 41)

Sun, Moon, Eclipses; Planets

Subscripts:

$s\ldots$ concerning the Sun, $m\ldots$ Moon,
$u\ldots$ shadow e.g. $r_u\ldots$ apparent radius of the shadow at the Moon's distance

$a\ldots$ ahargaṇa, i.e. days elapsed since epoch

$\bar{\lambda}\ldots$ mean longitude, $\overline{\Delta\lambda}\ldots$ mean synodic arc

$\theta = \lambda - \bar{\lambda}\ldots$ equation of center

$v\ldots$ velocity (usually in degrees per day)

$\lambda_{\mathrm{c}}\ldots$ longitude of the Moon's ascending node

$i\ldots$ inclination of the lunar orbit with respect to the ecliptic

$\beta\ldots$ lunar latitude

$p_0\ldots$ horizontal parallax, $p\ldots$ total parallax

$p_\lambda, p_\beta\ldots$ components of parallax (also lunar – solar)

Units of Time and Distance

Letters in raised position:

$d\ldots$ days $\tau\ldots$ tithis

$n\ldots$ nāḍīs $vin\ldots$ vināḍīs $\mu\ldots$ muhūrtas

y… years, sidereal or tropical, depending on context

m… months, usually mean synodic months

$z\ldots$ zodiacal signs, 30° each

na… nakṣatras 13;20° each

Linear Zigzag Functions

$m\ldots$ minimum $M\ldots$ maximum

$d\ldots$ constant difference (absolute value)

$\Delta = M - m\ldots$ amplitude,

$\mu = 1/2\,(m+M)\ldots$ mean value

$P = 2\Delta/d\ldots$ period $P = \Pi/Z$

4. Index of Parameters

Decimal

Lexicographically arranged; the integers from 1 to 59 are only listed in the sexagesimal index. For references see under the sexagesimal equivalent.

3200 = 53,20
335 = 5,35
3438 = 57,18
364 220 = 1,41,10,20
365 = 6,5
373 = 6,13
378 = 6,18
38 100 = 10,35,0
38 640 = 10,44,0
390 = 6,30
393 = 6,33
399 = 6,39

408 = 6,48
428 = 7,8
4 320 000 = 20,0,0,0
433 232 = 2,0,20,32
43 831 = 12,10,31
442 = 7,22
488 219 = 2,15,36,59

514 = 8,34
517 080 = 2,23,38,0
53 433 336 = 4,7,22,35,36
5347 = 1,29,7
54 787 = 15,13,7
56 266 = 15,37,46
57 753 336 = 4,27,22,35,36
583 = 9,43
584 = 9,44

60 = 1,0
609 = 10,9
6 313 219 = 29,13,40,19
6329 = 1,45,29
66 389 = 18,26,29
670 217 = 3,6,10,17
68 550 = 19,2,30
687 = 11,27
692 = 11,32
698 = 11,38

70 = 1,10
702 = 11,42
7 022 388 = 32,30,39,48
703 = 11,43
72 = 1,12
75 = 1,15
768 = 12,48
780 = 13,0

80 = 1,20
800 = 13,20
81 = 1,21
879 = 14,39
8797 = 2,26,37
88 = 1,28

900 = 15,0
900 000 = 4,10,0,0
927 see 1,55;52,30
97 = 1,37
9761 = 2,42,41
99 = 1,39

Sexagesimal

Lexicographically arranged, initial zeros being ignored. References are to chapter and verse in the commentary. The topics mentioned are not intended to give more than the general area to which a parameter belongs.

1,0	Venus, node XVI,12–14 (Table 22)
1,0	Mercury, ecliptic arc XVII,38–40
1,0	Saturn, manda epicycle XVI,12–14 (Table 22)
1;0,59,0,59	tithi XII,1 (p. 81 (2b))
1,1	tithi XII,1 (p. 80 (1))
1,2	months in 5 years XII,1
1,4,24	lunar diameter IX,15–16 (p. 72 (3))
1,4,28;11	solar diameter IX,15–16 (p. 72 (3))
1,4,30,26	lunar nodes, number of rotations in Mahāyuga I,14 (Table 1) XVI,1–9
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